Reports of the Department of Geodetic Science

Report No. 235

ON SIMILARITY TRANSFORMATIONS

AND

GEODETIC NETWORK DISTORTIONS BASED ON DOPPLER SATELLITE OBSERVATIONS

by

Alfred Leick and Boudewijn H.W. van Gelder

Prepared for the

National Aeronautics and Space Administration
Washington, D.C.

Grant No. NGR 36-008-204
OSURF Project No. 3820-A1

The Ohio State University
Research Foundation
Columbus, Ohio 43212

December, 1975

PREFACE AND ACKNOWLEDGEMENTS

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, The Ohio State University and is under the technical direction of Mr. James P. Murphy, Special Programs, Office of Applications, Code ES, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

The authors wish to express their appreciation to Mr. B.K. Meade of the National Ocean Survey for providing the Doppler station positions and the geodetic data of the United States; Mr. A.G. Bomford of the Department of Minerals and Energy, Canberra, for providing the data concerning Australia; and Messrs. K.I. Daugherty and R.P. Peat of the Defense Mapping Agency Topographic Center for providing the observational data for South America.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

The purpose of this report is twofold: (a) some theoretical contributions are given to the transformation models as used in geodesy to transform two sets of coordinates, (b) distortions in geodetic networks are investigated based on those transformation models.

Each section of this report can be considered as an entity in itself. At the end of each section separate conclusions are given. Specifically, Section 2 presents a review of the commonly used transformation models. Section 3 provides an interpretation of most of the models. It is shown that the translation components as computed from the so-called "Molodenskii model" for seven transformation parameters must not be interpreted as geometric vectors between the origins of the two coordinate systems involved. Only Bursa's model permits such interpretation. It is further shown that as for direct transformations both the Bursa and Molodenskii models give the same results, e.g., the same station coordinates and same variance-covariance matrix. It is also shown that the parameters as computed from both models differ only in the translation components. An expression is given to exactly convert one set of translation vectors to the other. Section 4 discusses pitfalls in partial solutions. It has been proposed in several publications (Bursa, 1967 and Kumar, 1972) to compute the rotation angles and the scale factor separately from chord and direction comparisons by using them in all combinations. Based on the method of eliminating parameters, it is shown that these methods are wrong as far as the selection of chords and/or directions are concerned. They yield parameters which

are close to those computed correctly but their standard deviations are much too optimistic. It is shown that only as many chords and/or directional elements can be used in the computation as are needed to completely determine the size or shape of the polyhedron implied in the set of Cartesian coordinates. Each additional element causes the normal matrix to be singular provided that all correlations between the chords are used.

Section 5 gives a number of tables and maps indicating distortions in the NAD 27, Precise Traverse M-R '72, AUS, and SAD 69 geodetic datums. The residuals of the coordinates are scanned for systematic patterns after transforming each geodetic system to the NWL9D Doppler system. Also, an attempt is made to show scale distortions in the NAD 27.

2. REVIEW OF SIMILARITY TRANSFORMATION MODELS

2.1 Similarity Transformation Model 1 (Bursa)

This transformation model is usually referred to as the Bursa model. Theoretical treatments of this model are given in (Veis, 1960; Bursa, 1962; Wolf, 1963; and Badekas, 1969).

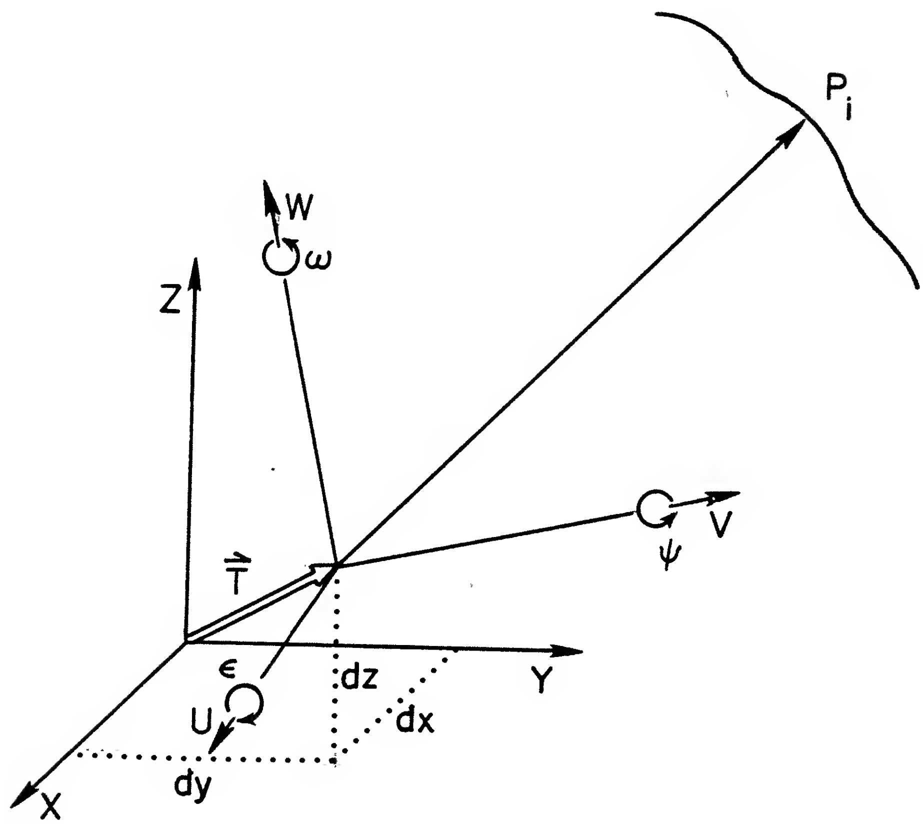


Fig. 2-1. Similarity Transformation Model 1 (Bursa)

The following coordinate systems are considered:

Average Terrestrial coordinate system $(X) = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. This is the coordinate system which is assumed to coincide with the system defined by the tracking station coordinates as obtained from satellite geodesy. It is defined as follows:

(a) z-axis directed toward the average north terrestrial pole as defined by the International Polar Motion Service (IPMS) commonly known as the Conventional International Origin (CIO).

(b) xz plane parallel to the mean Greenwich astronomic meridian as defined by the Bureau International de l'Heure (BIH).

Geodetic Datum (U) $= \begin{bmatrix} u \\ v \\ w \end{bmatrix}$. This is the coordinate system of the geodetic network. First, orthometric heights combined with geoid heights are used to form heights above the ellipsoid. These are combined with latitudes and longitudes of the horizontal network to form triplets of ellipsoidal coordinates which can then be converted along with their variance-covariance matrix into three-dimensional Cartesian coordinates.

The transformation equation expressed in the (X) system is readily seen from Fig. 2-1:

$$F \equiv \vec{T} + (1 + \Delta) R \vec{U} - \vec{X} = 0 \qquad (2\text{-}1)$$

where

$\vec{T}$ denotes the translation vector between the origins of the two systems in the (X) system

$1+\Delta$ denotes the scale factor between the systems

R is the product of three consecutive orthogonal rotations around the axes of (U):

$$R = R_W(\omega) R_V(\psi) R_U(\varepsilon) \qquad (2\text{-}2)$$

Using the conventional definitions of rotation matrices, one can write

$$R_U(\varepsilon) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{bmatrix} \qquad (2\text{-}3)$$

$$R_V(\psi) = \begin{bmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix} \tag{2-4}$$

$$R_W(\omega) = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-5}$$

and

$$R = \begin{bmatrix} \cos\psi\ \cos\omega & \cos\varepsilon\ \sin\omega + \sin\varepsilon\ \sin\psi\ \cos\omega & \sin\varepsilon\ \sin\omega - \cos\varepsilon\ \sin\psi\ \cos\omega \\ -\cos\psi\ \sin\omega & \cos\varepsilon\ \cos\omega - \sin\varepsilon\ \sin\psi\ \sin\omega & \sin\varepsilon\ \cos\omega + \cos\varepsilon\ \sin\psi\ \sin\omega \\ \sin\psi & -\sin\varepsilon\ \cos\psi & \cos\varepsilon\ \cos\psi \end{bmatrix} \tag{2-6}$$

The angles ε, ψ, ω are positive for counter-clockwise rotations about the respective u, v, w axes as viewed from the end of the positive axis (in case of right-handed coordinate systems).

Equation (2-1) defines the relation between the two systems in terms of seven parameters, e.g., three translations $\vec{T} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$; the scale factor $(1 + \Delta)$; the rotation angles ε, ψ, ω.

They are solved for in a least squares solution. The Cartesian coordinates of both systems are taken as observations. Equation (2-1) forms the mathematical model (Uotila, 1967)

$$F(L_a, X_a) = 0 \tag{2-7}$$

or

$$F(L_b + V, X_o + X) = 0 \tag{2-8}$$

where

L_a denotes the adjusted observation,

X_a the adjusted parameters,

L_b the observations,

X_o the approximate parameters,

V the residuals,

X the parameters solved for.

Since the rotations between systems (X) and (U) are small, it is permissible to simplify equation (2-6) as

$$R \approx I + Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \omega & -\psi \\ -\omega & 0 & \varepsilon \\ \psi & -\varepsilon & 0 \end{bmatrix} \tag{2-9}$$

Substituting (2-9) into (2-1) and neglecting second-order terms in scale Δ and rotations (ε, ψ, ω) and their products, the model can be written as

$$F \equiv \vec{T} + \vec{U} + \Delta\vec{U} + Q\vec{U} - \vec{X} = 0 \tag{2-10}$$

Each point P_i yields one such equation. The model (2-10) can now be linearized and the usual adjustment procedure

$$V^T PV = \min, \tag{2-11}$$

subject to the condition

$$BV + AX + W = 0 \tag{2-12}$$

applied, where

$$B = \frac{\partial F}{\partial L_a}$$

$$A = \frac{\partial F}{\partial X_a}$$

$$W = F(L_b, X_o)$$

and P is the weight matrix. Each point contributes three equations to the equation system (2-12), e.g., for point P_i, taking $X_0 \equiv 0$, one has

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & \vdots & -1 & 0 & 0 \\ 0 & 1 & 0 & \vdots & 0 & -1 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 0 & -1 \end{bmatrix}}_{B_i} \underbrace{\begin{bmatrix} V_u \\ V_v \\ V_w \\ V_x \\ V_y \\ V_z \end{bmatrix}_i}_{V_i} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & u & v & -w & 0 \\ 0 & 1 & 0 & v & -u & 0 & w \\ 0 & 0 & 1 & w & 0 & u & -v \end{bmatrix}_i}_{I \qquad A_i} \underbrace{\begin{bmatrix} dx \\ dy \\ dz \\ \Delta \\ \omega \\ \psi \\ \varepsilon \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} u - x \\ v - y \\ w - z \end{bmatrix}_i}_{W_i} = 0 \tag{2-13}$$

2.2 Similarity Transformation Model 2 (M-Badekas)

This model is described in (Badekas, 1969) and is attributed to Molodenskii, presumably referring to the treatment in (Molodenskii et al., 1962). However, the transformation as described in (Molodenskii et al., 1962) is based on differential transformation equations. The first vector interpretation of these differential equations is Badekas'. For a more detailed description of similarities and differences between the original Molodenskii formulas and Badekas' interpretation of these equations, the reader is referred to (Soler, 1976).

In the following sections, the Badekas interpretation of the Molodenskii formulas will be referred to as the M-Badekas model. It has been customary in the geodetic literature to derive Molodenskii's transformation equation from geometric interpretation of a figure which is similar to Fig. 2-1. At this stage we purposely postpone such interpretation to section 3.2.1 and simply state Molodenskii's transformation model as given in the literature (e.g., Badekas, 1969; Krakiwsky and Thomson, 1974). It is expressed in the satellite system (X) as follows:

$$F \equiv \vec{T} + \vec{U}_0 + (1 + \Delta)\, R(\vec{U}_i - \vec{U}_0) - \vec{X}_i = 0 \tag{2-14}$$

where

subscript i denotes the point P_i under consideration

U_0 is the positions vector of the initial point of the geodetic datum in the $\vec{U}$ system, but the <u>assumption</u> is made that this vector is identical to the same vector in a geodetic system which is already parallel to the Average Terrestrial system.

All other notation is the same as in equation (2-1).

If one follows the same procedure as described for the previous model, that is, omitting second-order terms in scale and rotation, and their products, the model (2-14) becomes

$$F \equiv \vec{T} + \vec{U} + \Delta(\vec{U} - \vec{U}_0) + Q(\vec{U} - \vec{U}_0) - \vec{X} = 0 \qquad (2\text{-}15)$$

Equation (2-15) is the mathematical model for a least squares solution. Each point contributes three condition equations of the following form:

$$\begin{array}{c}B_i\\ \begin{bmatrix}1 & 0 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & -1\end{bmatrix}\end{array}
\begin{array}{c}V_i\\ \begin{bmatrix}V_u\\ V_v\\ V_w\\ V_x\\ V_y\\ V_z\end{bmatrix}_i\end{array}
+
\begin{array}{c}I \qquad\qquad\qquad\qquad A_{Mi}\\ \left[\begin{array}{ccc|c|ccc}1 & 0 & 0 & u - u_0 & v - v_0 & -(w - w_0) & 0\\ 0 & 1 & 0 & v - v_0 & -(u - u_0) & 0 & w - w_0\\ 0 & 0 & 1 & w - w_0 & 0 & u - u_0 & -(v - v_0)\end{array}\right]\end{array}$$

$$\times \begin{array}{c}X\\ \begin{bmatrix}dx\\ dy\\ dz\\ \Delta\\ \omega\\ \psi\\ \varepsilon\end{bmatrix}\end{array}
+
\begin{array}{c}W_i\\ \begin{bmatrix}u - x\\ v - y\\ w - z\end{bmatrix}_i\end{array}
= 0 \qquad (2\text{-}16)$$

taking the approximate parameters $X_0 \equiv 0$.

2.3 Similarity Transformation Model 3 (Veis)

This model, introduced by Veis (1960), uses the same rotation point (initial point) as Model 2, but the rotations are about different axes (U_2). The U_2 axis is tangent to the geodetic meridian with positive direction toward the south; the V_2 axis is perpendicular to the meridian plane and it is positive eastward. Finally, the W_2 axis is along the geodetic normal with its positive direction upward, forming a right-handed system with U_2 and V_2. Similarly to equation (2-14), one obtains

$$F \equiv \vec{T} + \vec{U}_0 + (1 + \Delta)\, M(\vec{U}_i - \vec{U}_0) - \vec{X}_i = 0 \qquad (2\text{-}17)$$

If (η, ξ, α) denote positive rotations about the U_2, V_2, W_2 axes and ϕ_0, λ_0, h_0 the geodetic coordinates of the initial point, the M matrix is (Veis, 1960; Badekas, 1969):

$$M = R_3^T(\lambda_0)\, R_2^T(90 - \phi_0)\, R(\eta, \xi, \alpha)\, R_2(90 - \phi_0)\, R_3(\lambda_0) \qquad (2\text{-}18)$$

where R denotes the rotation matrices as in equations (2-3) - (2-6). If the rotation η, ξ, α are differentially small, one obtains

$$M = \begin{bmatrix} 1 & \begin{array}{l}\alpha \sin\phi_0 \\ -\eta\cos\phi_0\end{array} & \begin{array}{l}-\alpha\cos\phi_0\sin\lambda_0 \\ -\xi\cos\lambda_0 \\ -\eta\sin\phi_0\sin\lambda_0\end{array} \\ \begin{array}{l}-\alpha\sin\phi_0 \\ +\eta\cos\phi_0\end{array} & 1 & \begin{array}{l}\alpha\cos\phi_0\cos\lambda_0 \\ -\xi\sin\lambda_0 \\ +\eta\sin\phi_0\cos\lambda_0\end{array} \\ \begin{array}{l}\alpha\cos\phi_0\sin\lambda \\ +\xi\cos\lambda_0 \\ +\eta\sin\phi_0\sin\lambda_0\end{array} & \begin{array}{l}-\alpha\cos\phi_0\cos\lambda_0 \\ +\xi\sin\lambda_0 \\ -\eta\sin\phi_0\cos\lambda_0\end{array} & 1 \end{bmatrix} \qquad (2\text{-}19)$$

If again second-order terms in scale and rotations, and their products, are deleted, the model becomes

$$F = \vec{T} + U_0 + (1 + \Delta) M_1(\vec{U}_i - \vec{U}_0) - \vec{X}_i = 0 \tag{2-20}$$

where

$$M_1 = M - I \tag{2-21}$$

Thus the only difference between the M-Badekas and Veis models is that the rotations in the Veis model are familiar quantities; for example, a rotation about the W_2 axis corresponds to a rotation in azimuth. The three rotations ξ, η, α are related to the rotations ε, ψ, ω of Bursa's or M-Badekas' model as follows:

$$\begin{bmatrix} \alpha \\ \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \sin \phi_o & \cos \phi_o \sin \lambda_o & \cos \phi_o \cos \lambda_o \\ 0 & \cos \lambda_o & -\sin \lambda_o \\ -\cos \phi_o & \sin \phi_o \sin \lambda_o & \sin \phi_o \cos \lambda_o \end{bmatrix} \begin{bmatrix} \omega \\ \psi \\ \varepsilon \end{bmatrix} \tag{2-22}$$

Also, if $\Sigma_{\alpha,\xi,\eta}$ and $\Sigma_{\omega,\psi,\varepsilon}$ are the variance-covariance matrices in the two cases, then the principle of propagation of errors gives

$$\Sigma_{\alpha,\xi,\eta} = G\, \Sigma_{\omega,\psi,\varepsilon}\, G' \tag{2-23}$$

where

$$G = \begin{vmatrix} \sin \phi_o & \cos \phi_o \sin \lambda_o & \cos \phi_o \cos \lambda_o \\ 0 & \cos \lambda_o & -\sin \lambda_o \\ -\cos \phi_o & \sin \phi_o \sin \lambda_o & \sin \phi_o \cos \lambda_o \end{vmatrix} \tag{2-24}$$

2.4 Similarity Transformation Model 4 (Vanicek-Wells)

This model was introduced by Vanicek and Wells (1974). It has distinct differences with respect to the models discussed previously. The salient features of the model are:

(a) The fundamental system of reference is taken as the Average Terrestrial system (X).

(b) The satellite system (X') is assumed to have a different orientation than the Average Terrestrial system (X), i.e., three rotation angles $(\omega,\psi,\varepsilon)$ are needed to bring (X') into the (X) system.

(c) The assumption is made that the geodetic coordinate system (U) is a fixed framework invariant with respect to geodetic network adjustment. Proceeding from this assumption and the classical definition of a geodetic coordinate system, it is shown by Vanicek and Wells (1974) that in the transformation between such a geodetic system and the average terrestrial system the three rotation angles are dependent, being functions of a single rotation Ω of the geodetic system around the ellipsoidal normal at the initial point (ϕ_o, λ_o)

Expressing all the vectors in the Average Terrestrial system

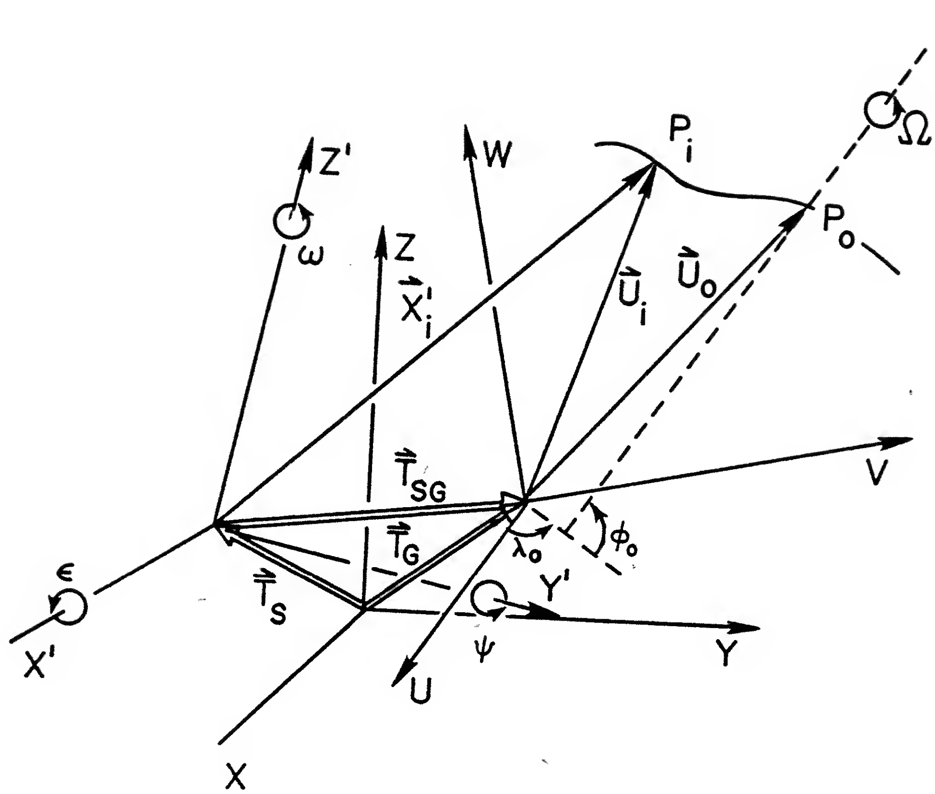
$$\vec{T}_S + R(\omega,\psi,\varepsilon)\ \vec{X}'_j = T_G + R(\Omega)\ \lambda\ (U_i) \tag{2-25}$$

where $R(\omega,\psi,\varepsilon)$ is the rotation matrix for small rotations as given in equation (2-9), ω,ψ,ε being the rotations around the Z',Y',X' axes, respectively. Introducing

$$R = I + Q$$

$$\lambda = 1 + \Delta$$

$$T_{SG} = T_G - T_S$$



(X') Satellite system
(X) Average Terrestrial system
(U) Geodetic system
ϕ_o, λ_o geodetic coordinates of the initial point P_o

Fig.2-2. Similarity Transformation Model 4 (Vanicek-Wells)

and deleting the term $Q\Delta(U_i)$, equation (2-25) can be given in the form

$$Q(\omega,\psi,\varepsilon)\vec{X}_j' - Q(\Omega)(\vec{U}_i) - T_{SG} - \Delta(\vec{U}_i) = \vec{U}_i - \vec{X}_i' \tag{2-26}$$

Substituting $Q(\Omega)(\vec{U}_i)$ by $Q(\Omega)\vec{X}_i$ and $\Delta\vec{U}_i$ by $\Delta\vec{X}_i$, which introduces an error of less than 1 cm (Wells and Vanicek, 1975), one finally arrives at

$$[Q(\omega,\psi,\varepsilon) - Q(\Omega) - \Delta]\vec{X}_i' - T_{SG} + (\vec{X}_i' - U_i) = 0 \tag{2-27}$$

Equation (2-27) is the mathematical model for a least squares solution with the following unknown parameters:

(1) the rotation angles ω, ψ, ε around the respective Z', Y', X' axes

(2) the rotation Ω of the geodetic system around the normal through the initial point. The unit vector in the direction of the ellipsoid normal at the initial point is

$$\vec{n} = (\cos\phi_o \cos\lambda_o, \cos\phi_o \sin\lambda_o, \sin\phi_o)^T \tag{2-28}$$

If $(\varepsilon_G, \psi_G, \omega_G)$ denote the U,V,W components of the rotation Ω , one can write

$$(\varepsilon_G, \psi_G, \omega_G) = \Omega(\cos\phi_o \cos\lambda_o, \cos\phi_o \sin\lambda_o, \sin\phi_o) \tag{2-29}$$

and by using equation (2-9)

$$Q(\Omega) = \begin{bmatrix} 0 & \Omega\sin\phi_o & -\Omega\cos\phi_o \sin\lambda_o \\ -\Omega\sin\phi_o & 0 & \Omega\cos\phi_o \cos\lambda_o \\ \Omega\cos\phi_o \sin\lambda_o & -\Omega\cos\phi_o \cos\lambda_o & 0 \end{bmatrix} \tag{2-29}$$

(3) the translation components T_{SG} of the geodetic system

(4) the scale difference Δ.

There are eight unknown parameters which require coordinates at least at three stations for a solution. In order to avoid singularities one needs more than one geodetic system. Since each geodetic datum increases the number of unknowns by five, at least two stations are required on each geodetic datum.

3. INTERPRETATION OF SIMILARITY TRANSFORMATION MODELS

3.1 Linear Transformation Models

In order to systematically discuss all the transformation models described in section 2, they are compared to a general linear transformation model which will allow drawing conclusions as to their independence from the effects of linearization, neglecting terms of the second order, correlation between station coordinates, etc.

The most general linear transformation between two sets of coordinates of points, given in two coordinate systems (X) and (U) is the affine transformation α:

$$\vec{X} = A\vec{U} + \vec{A}_0 \tag{3-1}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} a_{10} \\ a_{20} \\ a_{30} \end{bmatrix} \tag{3-2}$$

In principle, A and A_0 are arbitrary matrices, with some exceptional cases (e.g., A being a null matrix) which need not be considered here as they do not have useful applications in geodesy. Note that $A_0 = (a_{10}, a_{20}, a_{30})^T$ can be visualized as a translation in the (X) system.

A theorem (Modenov and Parkhomenko, 1961) states that any affine transformation α in Euclidean space is the product of three compressions onto three mutually perpendicular planes ζ and an orthogonality transformation ω:

$$\alpha = \zeta\omega = \zeta_3 \zeta_2 \zeta_1 \omega \tag{3-3}$$

In other words, the 12 parameters $(a_{11}, \ldots, a_{30})$ which describe the affine transformation α can be split up in a unique way: 6 parameters for

the orthogonality transformation ω (3 parameters for translation and 3 parameters for rotation) and 6 parameters for describing the scaling transformation ζ (3 scale parameters along three perpendicular axes of which the orientation is determined by 3 other parameters).

If the general affine transformation α (12 parameters) is reduced to an orthogonal transformation ω (6 parameters), the six well-known conditions imposed on the α transformation are:

$$\begin{aligned}
a_{11}a_{12} + a_{21}a_{22} + a_{31}a_{32} &= 0 \\
a_{11}a_{13} + a_{21}a_{23} + a_{31}a_{33} &= 0 \\
a_{12}a_{13} + a_{22}a_{23} + a_{32}a_{33} &= 0 \\
a_{11}^2 + a_{21}^2 + a_{31}^2 &= 1 \\
a_{12}^2 + a_{22}^2 + a_{32}^2 &= 1 \\
a_{13}^2 + a_{23}^2 + a_{33}^2 &= 1
\end{aligned} \tag{3-4}$$

An orthogonality transformation ω might be thought of as the result of a translation T and a rotation R. Two versions can be considered:

<u>A</u>. $$\vec{X} = R\vec{U} + \vec{T} \tag{3-5}$$

with

$$R = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix} \tag{3-6}$$

which can be viewed with the imposed conditions (3-4) as the result of three consecutive rotations around the U,V and W axes by the angles ε, ψ and ω:

$$R = R_W(\omega)\ R_V(\psi)\ R_U(\varepsilon) \tag{3-7}$$

The matrices $R_U(\varepsilon)$, $R_V(\psi)$, $R_W(\omega)$ and R are given by (2-3), (2-4), (2-5) and (2-6).

$\vec{T}$ might be interpreted as three translations along the X, Y and Z axes:

$$\vec{T} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \tag{3-8}$$

B. $$\vec{X} = R(\vec{U} + \vec{T}) \tag{3-9}$$

R is identical to (2-6) with the exception that the three rotations ε, ψ, ω are not around the U, V, and W axes but around the (U + dU), (V + dV) and (W + dW) axes. Consequently, (3-7) becomes

$$R = R_{W+dW}(\omega)R_{V+dV}(\psi)R_{U+dU}(\varepsilon) \tag{3-10}$$

$\vec{T}$ represents three translations along the U, V and W axes,

$$\vec{T} = \begin{bmatrix} du \\ dv \\ dw \end{bmatrix} \tag{3-11}$$

In geodesy a transformation more restricted than the general affine transformation α (12 parameters) but more relaxed than the orthogonality transformation ω (6 parameters) is being used. This is the "7 parameter similarity transformation σ," the seventh parameter being the scale difference between the X and U coordinate systems. This transformation with coefficient λ (scale factor) with center O is known as a homothetic transformation β.

A theorem (Modenov and Parkhomenko, 1961) states that any similarity transformation σ with coefficient λ can be represented as the product of a

homothetic transformation β, with coefficient λ and prescribed center 0, and an orthogonality transformation ω as follows:

$$\sigma = \beta \omega \tag{3-12}$$

It can be shown that β and ω are uniquely determined by λ and 0 and by the requirement that ω be an orthogonal transformation.

It can be shown that the scale parameter λ of the homothetic transformation is a special case of the 3 scale parameter transformation:

$$\beta = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{3-13}$$

which in its turn is a special case of the general compression transformation ζ:

$$\zeta = \beta = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \tag{3-14}$$

In this general matrix β, the eigenvalues form the three different scale factors (compressions) onto the 3 perpendicular planes of which the normal vectors are the eigenvectors of matrix β.

Four versions of the similarity transformation σ (7 parameters) may be written as

A. A1 $\vec{X} = \lambda R\vec{U} + \vec{T}$ (3-15)

A2 $\vec{X} = \lambda(R\vec{U} + \vec{T})$ (3-16)

B. B1 $\vec{X} = \lambda R(\vec{U} + \vec{T})$ (3-17)

B2 $\vec{X} = R(\lambda\vec{U} + \vec{T})$ (3-18)

It is readily seen that the first three models discussed in section 2 are in version A1.

The following relations may be found using a slightly different notation for each model:

$$\vec{X} = \lambda R\vec{U} + \vec{T} \qquad \text{(A1)} \tag{3-19}$$

Model 1 (Bursa)

From equation (2-1):

$$\begin{aligned} \lambda &\equiv 1 + \Delta_B \\ R &\equiv R_B \\ \vec{T} &\equiv \vec{T}_B \end{aligned} \tag{3-20}$$

yielding

$$B_7: \quad \vec{X} = (1 + \Delta_B)\ R_B\vec{U} + \vec{T}_B \quad . \tag{3-21}$$

Model 2 (M-Badekas)

From equation (2-14):

$$\begin{aligned} \lambda &\equiv 1 + \Delta_M \\ R &\equiv R_M \\ \vec{T} &\equiv \vec{T}_M + \vec{U}_0 \\ \vec{U} &\equiv \vec{U} - \vec{U}_0 \end{aligned} \tag{3-22}$$

yielding

$$M_7: \quad \vec{X} = (1 + \Delta_M)\ R_M(\vec{U} - \vec{U}_0) + \vec{U}_0 + \vec{T}_M \tag{3-23}$$

Model 3 (Veis)

From equation (2-17):

$$\begin{aligned} \lambda &\equiv (1 + \Delta_V) \\ R &\equiv R_V \\ \vec{T} &\equiv \vec{T}_V + \vec{U}_0 \\ \vec{U} &\equiv \vec{U} - \vec{U}_0 \end{aligned} \tag{3-24}$$

yielding

$$V_7: \qquad \vec{X} = (1 + \Delta_V)\, R_V(\vec{U} - \vec{U}_0) + \vec{U}_0 + \vec{T}_V \tag{3-25}$$

Model 4 (Vanicek-Wells)

From equation (2-25):

$$\lambda \equiv 1 + \Delta_W$$

$$R \equiv R_W \qquad\qquad R \equiv R'_W$$

$$\vec{T} = \vec{T}_W \qquad\qquad \vec{T} \equiv \vec{T}'_W$$

yielding

$$\vec{X} = \vec{T}_W + R_W\vec{X}' = \vec{T}'_W + (1 + \Delta_W)_j\, R'_{Wj}\, \vec{U}_j \tag{3-26}$$

In the following comparison of the models, Model 4 will have to be treated separately since it relates more than two coordinate systems to each other ($j = 2,r$ where r is number of geodetic datums).

3.2 Comparisons of Seven-Parameter Transformation Models

3.2.1 Model 1 (Bursa) vs. Model 2 (M-Badekas)

Recalling Model 1 (Eq. 3-21)

$$\vec{X}_i = (1 + \Delta_B)\, R_B\, \vec{U}_i + \vec{T}_B \tag{3-27}$$

and Model 2 (Eq. 3-23)

$$\vec{X}_i = (1 + \Delta_M)\, R_M(\vec{U}_i - \vec{U}_0) + \vec{U}_0 + \vec{T}_M \tag{3-28}$$

construct similarity transformation equations in terms of coordinate differences. Rewrite (3-27) and (3-28) with respect to the coordinate differences as computed from point 1 (using $\vec{X}_{i1} = \vec{X}_i - \vec{X}_1$ etc., point 1 is arbitrary)

$$\vec{X}_{i1} = (1 + \Delta_B)\, R_B\, \vec{U}_{i1} \tag{3-29}$$

$$\vec{X}_{i1} = (1 + \Delta_M)\, R_M\, \vec{U}_{i1} \tag{3-30}$$

It may be recognized that both models (3-29) and (3-30) are identical due to cancellations in (3-27) of $\vec{T}_B$ and in (3-28) of U_0 and $\vec{T}_M$. This simply means that Model 1 (Bursa) and Model 2 (M-Badekas) result in identical values for the scale factor and rotational parameters $(\varepsilon,\psi,\omega)$:

$$\begin{aligned} \Delta_B &\equiv \Delta_M = \Delta \\ R_B &\equiv R_M = R \end{aligned} \tag{3-31}$$

Both models may be rewritten as

Model 1: $$\vec{X}_i = (1+\Delta)\, R\, \vec{U}_i + \vec{T}_B \tag{3-32}$$

Model 2: $$\vec{X}_i = (1+\Delta)\, R(\vec{U}_i - \vec{U}_0) + \vec{U}_0 + \vec{T}_M \tag{3-33}$$

Since $\vec{X}_i$ and $\vec{U}_i$ are also identical in both models, the inclusion of U_0 in the second model has to be absorbed completely by $\vec{T}_M$. Compare both the translational vectors $\vec{T}_B$ and $\vec{T}_M$:

$$\begin{aligned} \vec{T}_B &= \vec{X}_i - (1+\Delta)\, R\, \vec{U}_i \\ \vec{T}_M &= \vec{X}_i - (1+\Delta)\, R(\vec{U}_i - \vec{U}_0) - \vec{U}_0 \end{aligned}$$

and form the difference:

$$\vec{T}_B - \vec{T}_M = \vec{U}_0 - (1+\Delta)\, R\, \vec{U}_0 \tag{3-34}$$

or

$$\vec{T}_B = \vec{T}_M + \vec{U}_0 - (1+\Delta)\, R\, \vec{U}_0 \tag{3-35}$$

The last equation shows that the simple geometrical interpretation of $\vec{T}_B$, as depicted in Fig. 2-1, namely the translation of the U (geodetic) system with respect to the X (satellite) system as measured in the X system, is not possible for $\vec{T}_M$.

One has to return to the original equation of Model 2 to see what is the cause of this loss of geometric significance. Rewrite (3-28) as

$$\vec{X}_i - \vec{U}_0 - \vec{T}_M = (1+\Delta_M)\, R_M(\vec{U}_i - \vec{U}_0) \tag{3-36}$$

and notice that in the right-hand side $\vec{U}_0$ denotes a vector (initial point of the geodetic datum) in the (U) system, whereas in the left-hand side $\vec{U}_0$ is assumed to be a vector in the (X) system. This incompatible use of the vector $\vec{U}_0$ is the cause of its difficult geometric interpretation (Fig. 3-1).

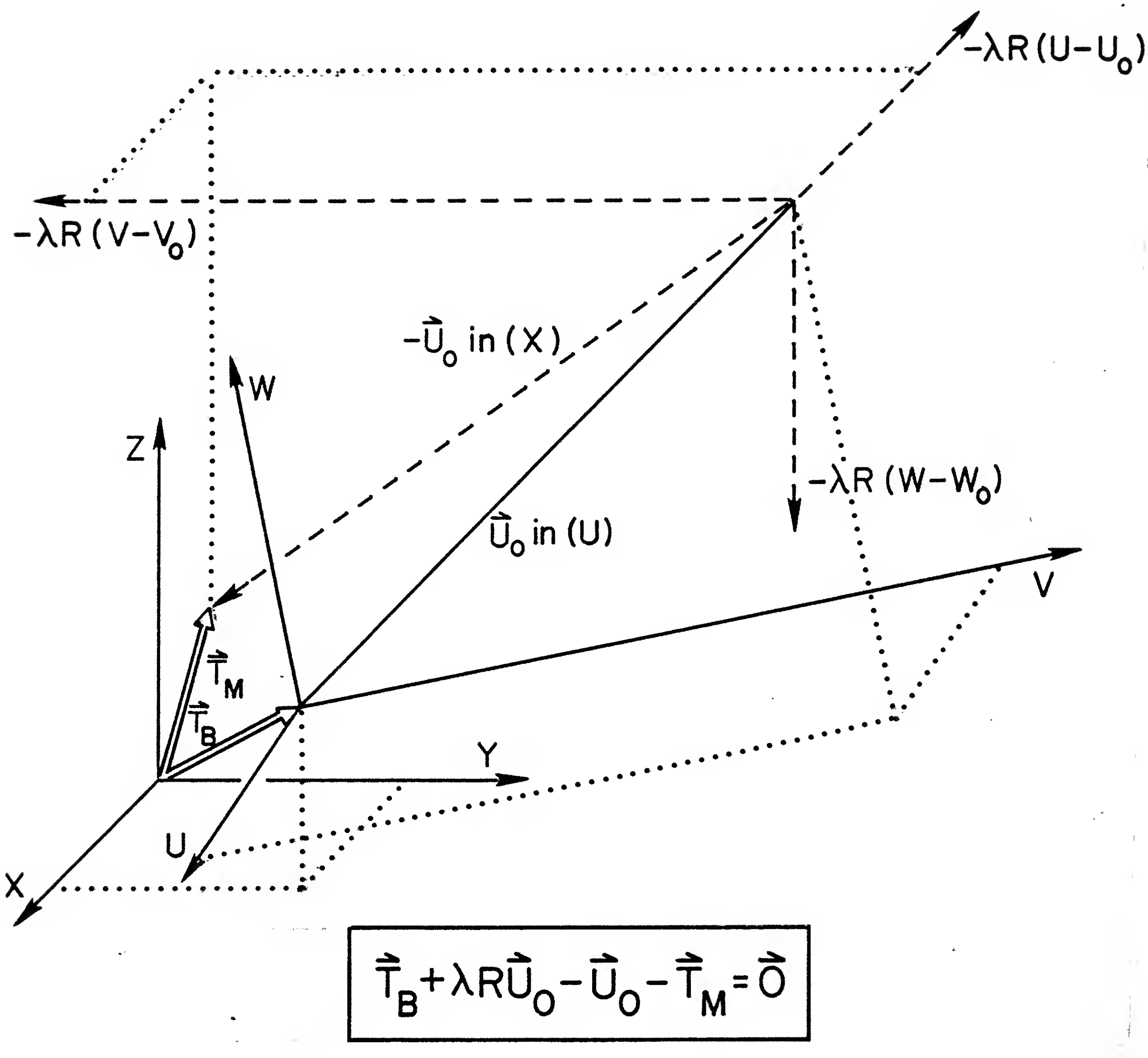


Fig. 3-1. Similarity Transformation Model 2 (M-Badekas)

Note that if $\vec{U}_0$ is correctly used at the left-hand side of (3-36), i.e., $(1 + \Delta_M)R_M\vec{U}_0$ instead of $\vec{U}_0$, one obtains Model 1:

$$\vec{X}_i - (1 + \Delta_M)\, R_M \vec{U}_0 - \vec{T}_M = (1 + \Delta_M)\, R_M\, (\vec{U}_i - \vec{U}_0) \tag{3-37}$$

or $$\vec{X}_i = (1 + \Delta_M)\, R_M (\vec{U}_i - \vec{U}_0) + (1 + \Delta_M)\, R_M \vec{U}_0 + \vec{T}_M$$

or $$\vec{X}_i = (1 + \Delta_M)\, R_M \vec{U}_i + \vec{T}_M \equiv (1 + \Delta_B)\, R_B \vec{U}_i + \vec{T}_B \tag{3-38}$$

This is exactly how the M-Badekas model can be derived from the Bursa model by assuming that $(1 + \Delta_M) R_M \vec{U}_0 \equiv \vec{U}_0$.

It has been argued that Model 2 reduces correlation between rotational and translational parameters which exists in Model 1 in case of small areas. The fact is that Model 2 does not minimize correlation but consists of completely different translational parameters which happen to have a smaller correlation with the rotational parameters (which are identical in both models).

It should be clear that given a set of points in two coordinate systems (U) and (X), the correlation for Model 1 as well as for Model 2 are fixed and cannot be altered (of course, we do not add constraints, observations, etc.). Moreover, both variance-covariance matrices of Model 1 and Model 2 are completely dependent and can be computed from each other by means of (3-35).

In addition, if the direct transformation is applied (given $\vec{U}_i$, $\Sigma_{\vec{U}_i}$ and 7 transformation parameters and their variance-covariance matrix, compute $\vec{X}$ and $\Sigma_{\vec{X}}$), <u>identical</u> $\vec{X}_i$'s and $\Sigma_{\vec{X}_i}$ will be obtained using Model 1 with highly correlated transformation parameters or Model 2 with almost uncorrelated transformation parameters.

This undermines the usage of Model 2 in two ways: (a) the reduction of the correlation between rotational and translation parameters is artificial and misleading, (b) the translation parameters of Model 2 have hardly any geometric significance.

An (academic ?) future for Model 2 is feasible only when one tries to answer the following question: "What choice of $\vec{U}_0$ minimizes the correlation between rotational and translative parameters?"

First of all, the geodetic interpretation of $\vec{U}_0$ (position vector of the initial point of the geodetic datum) in the model is irrelevant, i.e., the rotational and scale parameters (and their variance-covariance matrices) are completely independent of the choice of $\vec{U}_0$, as shown in (3-29) thru (3-31). This being the case, one might include $\vec{U}_0$ as an unknown parameter in the transformation model and solve for it in a "least squares" adjustment by minimizing not only V^TPV but also $\rho_{R,\vec{T}}$ which symbolically represents the covariance matrix between the rotational and translation parameters.

The uniqueness of any similarity transformation was already described by a theorem quoted in section 3.1. The coefficient and the center of the homothetic transformation (in Model 1 the center is $\vec{U} = \vec{0}$, in Model 2 the center is $\vec{U} = \vec{U}_0$) and the requirement that the remaining parameters form an orthogonal transformation uniquely define the similarity transformation. It means that the parameters of Model 1 and Model 2 are interdependent yielding the same results in the direct transformation. Moreover, if we write (3-32) and (3-33) in terms of the observed coordinates rather than the adjusted ones, we obtain

$$\text{Model 1:} \qquad \vec{X}_{ib} + \vec{V}_{X,B} = (1 + \Delta_a)\, R_a(\vec{U}_{ib} + \vec{V}_{U,B}) + \vec{T}_{B,a} \tag{3-39}$$

$$\text{Model 2:} \qquad \vec{X}_{ib} + \vec{V}_{X,M} = (1 + \Delta_a)\, R_a(\vec{U}_{ib} + \vec{V}_{U,M}) + \vec{T}_{M,a} \tag{3-40}$$

where b denotes observed values and a denotes adjusted values. The uniqueness theorem implies that the residuals in both models will be identical:

$$\vec{V}_{X,B} \equiv \vec{V}_{X,M} \tag{3-41}$$

$$\vec{V}_{U,B} \equiv \vec{V}_{U,M} \tag{3-42}$$

(3-41) and (3-42) among other characteristics in which Model 1 and Model 2 are interdependent will be shown below, using the least squares estimation procedure. Recalling the linearized form of Model 1 (Bursa)

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right]_i \begin{bmatrix} V_u \\ V_v \\ V_w \\ V_x \\ V_y \\ V_z \end{bmatrix}_i + \begin{bmatrix} 1 & 0 & 0 & u & v & -w & 0 \\ 0 & 1 & 0 & v & -u & 0 & w \\ 0 & 0 & 1 & w & 0 & u & -v \end{bmatrix}_i \begin{bmatrix} dx \\ dy \\ dz \\ \Delta \\ \omega \\ \psi \\ \varepsilon \end{bmatrix} + \begin{bmatrix} u - x \\ v - y \\ w - z \end{bmatrix}_i = 0 \tag{3-43}$$

or

$$B_B V_B + A'_B X_B + W_B = 0 \tag{3-44}$$

and of Model 2 (M-Badekas)

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right]_i \begin{bmatrix} V_u \\ V_v \\ V_w \\ V_x \\ V_y \\ V_z \end{bmatrix}_i + \begin{bmatrix} 1 & 0 & 0 & (u-u_0) & (v-v_0) & -(w-w_0) & 0 \\ 0 & 1 & 0 & (v-v_0) & -(u-u_0) & 0 & (w-w_0) \\ 0 & 0 & 1 & 0 & 0 & (u-u_0) & -(v-v_0) \end{bmatrix}_i$$

$$\begin{bmatrix} dx \\ dy \\ dz \\ \Delta \\ \omega \\ \psi \\ \varepsilon \end{bmatrix} + \begin{bmatrix} u - x \\ v - y \\ w - z \end{bmatrix}_i = 0 \tag{3-45}$$

or

$$B_M V_M + A'_M X_M + W_M = 0 \tag{3-46}$$

it may be recognized that

$$B_{B_i} \equiv B_{M_i} = B_i = [I \,\vdots\, -I]_i \tag{3-47}$$

$$W_{B_i} \equiv W_{M_i} = W_i \tag{3-48}$$

$$A'_{B_i} \equiv (I \,\vdots\, A_B)_i \tag{3-49}$$

$$A'_{M_i} \equiv (I \,\vdots\, A_M)_i = (I \,\vdots\, A_B - D)_i \tag{3-50}$$

where

$$D = \begin{bmatrix} u_o & v_o & -w_o & 0 \\ v_o & -u_o & 0 & w_o \\ w_o & 0 & u_o & -v_o \end{bmatrix} \tag{3-51}$$

If the X vector is partitioned into a translational part (X_1) and a scale plus rotational part (X_2) the linearized models (3-44) and (3-46) become

$$B_i V_{B_i} + (I \,\vdots\, A_B)_i \begin{bmatrix} X_{B_1} \\ \hline X_{B_2} \end{bmatrix} + W_i = 0 \tag{3-52}$$

$$B_i V_{M_i} + (I \,\vdots\, A_M)_i \begin{bmatrix} X_{M_1} \\ \hline X_{M_2} \end{bmatrix} + W_i = 0 \tag{3-53}$$

with

$$A_M = A_B - D \tag{3-54}$$

and i = 1, ..., r (r denotes the number of points to be transformed).

In the following section it will be proved that

(1) $X_{B_2} = X_{M_2}$

(2) $V_{B_i} = V_{M_i}$

(3) In the direct transformation identical results will be obtained for the newly transformed coordinates and their variance-covariance matrix (i.e., coordinates which were not used in the estimation procedure).

Equations (3-44) and (3-46) lead to the well-known expressions (Uotila, 1967)

$$X = -(A^TM^{-1}A)^{-1} A^TM^{-1}W \tag{3-55}$$

$$V = -P^{-1}B^TM^{-1} (AX + W) \tag{3-56}$$

$$\hat{\sigma}_0^2 = V^TPV/DF \tag{3-57}$$

where

$$M = BP^{-1}B^T \tag{3-58}$$

The standard notation which has been used in the equations above is explained subsequent to equation (2-12) in the previous section. For the derivation, using submatrices, partition the matrices of (3-52) and (3-53) as follows:

$$\underset{3r\ 6r}{B} = \begin{bmatrix} \underset{3\ 3}{I} & \underset{3\ 3}{-I} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \underset{3\ 3}{I} & \underset{3\ 3}{-I} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \underset{3\ 3}{I} & \underset{3\ 3}{-I} \end{bmatrix} \tag{3-59}$$

$$_{6r}V_1 = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_r \end{bmatrix} = \begin{bmatrix} V_{1U} \\ V_{1X} \\ V_{2U} \\ V_{2X} \\ \vdots \\ V_{rU} \\ V_{rX} \end{bmatrix} \quad \text{(3-60}$$

with

$$V_{iU} = \begin{bmatrix} v_{iu} \\ v_{iv} \\ v_{iw} \end{bmatrix} \qquad \text{and} \qquad V_{iX} = \begin{bmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \end{bmatrix} \tag{3-61}$$

$${}_{3r}A_{u=7} = \left[\begin{array}{c|c} {}_3I_3 & {}_3A_{1_4} \\ {}_3I_3 & {}_3A_{2_4} \\ \vdots & \vdots \\ {}_3I_3 & {}_3A_{r_4} \end{array}\right] \tag{3-62}$$

$${}_{3r}W_1 = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \end{bmatrix} = \begin{bmatrix} \vec{u}_1 - \vec{x}_1 \\ \vec{u}_2 - \vec{x}_2 \\ \vdots \\ \vec{u}_r - \vec{x}_r \end{bmatrix} \tag{3-63}$$

To simplify the derivation assume no correlation between the coordinates of various stations, that is, the weight coefficient matrix is of (3 x 3) block diagonal form

$${}_{6r}P^{-1}_{6r} = \frac{1}{\sigma_o^2} \begin{bmatrix} \bar{k}\,\Sigma_{U_1} & 0 & 0 & 0 & & & \\ 0 & \Sigma_{X_1} & 0 & 0 & & & \\ 0 & 0 & \bar{k}\,\Sigma_{U_2} & 0 & & & \\ 0 & 0 & 0 & \Sigma_{X_2} & & & \\ & & & & \ddots & & \\ & & & & & \bar{k}\,\Sigma_{U_r} & 0 \\ & & & & & 0 & \Sigma_{X_r} \end{bmatrix} \tag{3-64}$$

Comparing equations (3-43) and (3-45) reveals already that Model 1 and Model 2 differ in the design matrix A by a matrix of constants: D. One can write the A matrix of Model 2 (M-Badekas), using (3-62), as

$$A'_M = \begin{bmatrix} I & A_{M_1} \\ I & A_{M_2} \\ \vdots & \vdots \\ I & A_{M_r} \end{bmatrix} = \begin{bmatrix} I & A_{B_1} - D \\ I & A_{B_2} - D \\ \vdots & \vdots \\ I & A_{B_r} - D \end{bmatrix} \tag{3-65}$$

The derivation from this point is made for Model 2 (M-Badekas). One obtains at each of the following steps the expressions for Model 1 (Bursa) if one sets D = 0 and replaces X_M by X_B and V_M by V_B.

As a first step, the matrix M is obtained by (3-58).

$$M = \frac{1}{\sigma_0^2} [\bar{K} \not{\Sigma}_U + \not{\Sigma}_X] \tag{3-66}$$

Because of the block diagonal nature of $\not{\Sigma}_U$ and $\not{\Sigma}_X$, the inverse of M consists of (3 x 3) blocks. Denote each block $\bar{P}_i$. Then

$$M_i^{-1} = \bar{P}_i = \sigma_0^2 [k \not{\Sigma}_{U_i} + \not{\Sigma}_{X_i}]^{-1} \tag{3-67}$$

The normal matrix $A_M'^T M^{-1} A_M'$, after some matrix manipulation, will be:

$$A_M'^T M^{-1} A_M' = \left[\begin{array}{c|c} \Sigma \bar{P}_i & \Sigma \bar{P}_i A_{B_i} - \Sigma \bar{P}_i D \\ \hline \Sigma A_{B_i}^T \bar{P}_i - \Sigma D^T \bar{P}_i & \Sigma A_{B_i}^T \bar{P}_i A_{B_i} - 2\Sigma A_{B_i}^T \bar{P}_i D + \Sigma D^T \bar{P}_i D \end{array}\right] \tag{3-68}$$

Assume that summation is taken over r unless otherwise stated. In addition, drop the subindex B, i.e., $A_B \equiv A$.

The inverse of the normal matrix (3-68) can be obtained by using the general formulas for the inverse of partitioned matrices (Uotila, 1967).

If N is symmetric, and N_{11} and N_{22} are square matrices and of full rank, it is known that

$$N^{-1} = \begin{bmatrix} N_{11} & N_{12} \\ N_{12}^T & N_{22} \end{bmatrix}^{-1} = \left[\begin{array}{c|c} N_{11}^{-1} + N_{11}^{-1} N_{12} \bar{N}^{-1} N_{12}^T N_{11}^{-1} & - N_{11}^{-1} N_{12} \bar{N}^{-1} \\ \hline - \bar{N}^{-1} N_{12}^T N_{11}^{-1} & \bar{N}^{-1} \end{array}\right] \tag{3-69}$$

with

$$\bar{N} = N_{22} - N_{12}^T N_{11}^{-1} N_{12} \tag{3-70}$$

Using these expressions

$$(A_M'^T M^{-1} A_M')^{-1} = \left[\begin{array}{c|c} \begin{array}{c}(\Sigma\bar{P}_i)^{-1} + (\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i A_i - \Sigma\bar{P}_i D)S^{-1} \\ (\Sigma\bar{P}_i A_i - \Sigma\bar{P}_i D)^T(\Sigma\bar{P}_i)^{-1}\end{array} & -(\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i A_i - \Sigma\bar{P}_i D)S^{-1} \\ \hline -S^{-1}(\Sigma\bar{P}_i A_i - \Sigma\bar{P}_i D)^T(\Sigma\bar{P}_i)^{-1} & S^{-1} \end{array}\right] \tag{3-71}$$

where

$$S = \Sigma A_i^T \bar{P}_i A_i - (\Sigma\bar{P}_i A_i)^T(\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i A_i) \tag{3-72}$$

Carrying out the matrix manipulation indicated in (3-55), the expressions for the parameters $(X_1^T \mid X_2^T)$ become:

$$\begin{aligned} X_{M_1} = {} & \{(\Sigma\bar{P}_i)^{-1} + (\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i A_i)S^{-1}(\Sigma\bar{P}_i A_i)^T(\Sigma\bar{P}_i)^{-1}\}\ (\Sigma\bar{P}_i W_i) \\ & - (\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i A_i)^T\ S^{-1}(\Sigma A_i^T \bar{P}_i W_i) \\ & + D^T S^{-1}\ \{(\Sigma A_i^T P_i W_i) - (\Sigma\bar{P}_i A_i)^T(\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i W_i)\} \end{aligned} \tag{3-73}$$

$$X_{M_2} = S^{-1}\{(\Sigma A_i^T \bar{P}_i W_i) - (\Sigma\bar{P}_i A_i)^T(\Sigma\bar{P}_i)^{-1}(\Sigma\bar{P}_i W_i)\} \tag{3-74}$$

By inspecting (3-74), the expression for X_{M_2}, which represents the scale and rotations, is seen to be independent of the submatrix D, i.e., X_{M_2} is independent of the selection of the point of scaling and rotation. Even when this point is chosen at the origin of the U coordinate system, which happens in the case of Model 1 (Bursa), the same scale and rotational parameters are obtained. The presence of the submatrix D in the last term of X_{M_1} (3-73) indicates the dependency of the outcome of the translation parameters on the point of scaling and rotation. The last term of (3-73) is exactly the difference in translation vectors as obtained by Bursa (X_{B_1}, D = 0) and M-Badekas (X_{M_1}, D $\neq$ 0).

It is also easily shown that both methods yield the same residuals ($V_B = V_M$). To prove this, the product AX has to be invariant with respect to the models according to equations (3-44) and (3-46) (or 3-56), since in both models

$$V_B = V_M$$

if

$$B_B = B_M$$

$$W_B = W_M$$

and

$$A_B X_B = A X_M$$

The first two requirements are fulfilled directly by inspection of (3-43) and (3-45).

For point i

Model 1:

$$(I \;\; A_B)_i \begin{bmatrix} X_{B_1} \\ \hdashline X_{B_2} \end{bmatrix} = X_{B_1} + A_{B_i} X_{B_2} \tag{3-75}$$

Model 2:

$$(I \;\; A_M)_i \begin{bmatrix} X_{M_1} \\ \hdashline X_{M_2} \end{bmatrix} = X_{M_1} + A_{M_i} X_{M_2} \tag{3-76}$$

Using

$$A_{M_i} = A_{B_i} - D \tag{3-54}$$

(3-76) becomes

$$(I\ A_M)_i \begin{bmatrix} X_{M_1} \\ --- \\ X_{M_2} \end{bmatrix} = X_{M_1} + A_{B_i} X_{M_2} - D X_{M_2} \tag{3-77}$$

$$= X_{M_1} + A_{B_i} X_{B_2} - D X_{B_2} \tag{3-78}$$

From (3-73) and (3-74), one obtains

$$\begin{aligned} X_{M_1} &= X_{B_1} + D X_{M_2} \\ &= X_{B_1} + D X_{B_2} \end{aligned} \tag{3-79}$$

since $X_{M_2} = X_{B_2}$.

Substituting (3-79) into (3-78)

$$(I\ A_M)_i \begin{bmatrix} X_{M_1} \\ --- \\ X_{M_2} \end{bmatrix} = X_{B_1} + A_{B_i} X_{B_2}$$

which is identical to (3-75). Consequently, the proof that

$$A_B' X_B = A_M' X_M \tag{3-80}$$

verifies that $V_B = V_M$, which was proved in a different way prior to (3-41) and (3-42).

As a last step it is shown that both models are equivalent as applied in direct transformations. Equations (2-10) and (2-15) are the mathematical models for this transformation. Using the notation as in (3-62) and (3-65) they are written for the point P_j as

Bursa: $$\vec{X}_{B_j} = \vec{U}_j + A_{B_j}' X_B \tag{3-81}$$

$$\text{M-Badekas:} \qquad \vec{X}_{M_j} = \vec{U}_j + A'_{M_j} X_M \tag{3-82}$$

Because of the relation (3-80) it is readily seen that

$$\vec{X}_{B_j} = \vec{X}_{M_j} \tag{3-83}$$

holds. It is now only left to prove that both variance-covariance matrices are also equal, e.g.,

$$\not{\Sigma}\, \vec{X}_{B_j} = \not{\Sigma}\, \vec{X}_{M_j} \tag{3-84}$$

Differentiating equation (3-82) yields

$$d\vec{X}_{M_j} = \frac{\partial(A'_{M_j} X_M + \vec{U}_j)}{\partial X_M} dX_M + \frac{\partial(A'_{M_j} X_M + \vec{U}_j)}{\partial \vec{U}_j} d\vec{U}_j \tag{3-84}$$

The partial differentiation in the first term gives

$$\frac{\partial(A'_{M_j} X_M + \vec{U}_j)}{\partial X_M} = A'_{M_j} \tag{3-85}$$

In order to carry out the differentiation in the second term, it is rewritten as follows

$$\begin{aligned} A'_{M_j} X_M + \vec{U}_j &\equiv (I \mid \vec{U}_j - \vec{U}_o \mid Q_{(\omega,\psi,\varepsilon)}) \begin{bmatrix} \vec{X}_{1M} \\ \Delta \\ \vec{U}_j - \vec{U}_o \end{bmatrix} + \vec{U}_j \\ &= \vec{X}_{1M} + \Delta(\vec{U}_j - \vec{U}_o) + Q(\vec{U}_j - \vec{U}_o) + \vec{U}_j \end{aligned} \tag{3-86}$$

where Q is given in equation (2-9). Thus

$$\frac{\partial(A'_{M_j}\; X_M + \vec{U}_j)}{\partial\, \vec{U}_j} = I + \Delta I + Q = \overline{Q} \qquad (3.87)$$

Using (3-85) and (3-87) and the law of propagation of covariances, one obtains

$$\Sigma_{\vec{X}_{M_j}} = (A'_{M_j}\;\; \overline{Q}) \begin{bmatrix} \Sigma_{X_M} & 0 \\ 0 & \Sigma_{\vec{U}_j} \end{bmatrix} \begin{bmatrix} A'^{T}_{M_j} \\ \overline{Q}^T \end{bmatrix} = A'_{M_j}\; \Sigma_{X_M}\; A'^{T}_{M_j} + \overline{Q}\; \Sigma_{\vec{U}_j}\; \overline{Q}^T \qquad (3\text{-}88)$$

The second term is only a function of the scale and the rotation angles, and consequently is the same for both the Bursa and the M-Badekas models. It is easy to show that the first term satisfies the relation

$$A'_{M_j}\; \Sigma_{X_M}\; A'^{T}_{M_j} = A'_{B_j}\; \Sigma_{X_B}\; A'^{T}_{B_j} \qquad (3\text{-}89)$$

To prove this, recall that

$$\Sigma_{X_M} = (A'^{T}_{M}\; M^{-1}\; A'_M)^{-1} \qquad (3\text{-}90)$$

This matrix is already given in (3-71). After some elementary matrix multiplications as indicated in equation (3-88), one obtains an expression which is independent of the matrix D (3-51), which proves the relation (3-89). The complete expression for the covariance matrix of the transformed point P_j is

$$\begin{aligned} \Sigma_{\vec{X}_j} = {} & (\Sigma\overline{P}_i)^{-1} + (\Sigma\overline{P}_i)^{-1}(\Sigma\overline{P}_i A_i)\; S^{-1}(\Sigma\overline{P}_i A_i)^T(\Sigma\overline{P}_i)^{-1} \\ & - 2(\Sigma\overline{P}_i)^{-1}(\Sigma\overline{P}_i A_i)\; S^{-1} A_j^T + A_j\; S^{-1} A_j^T + \overline{Q}\; \Sigma_{U_j}\; \overline{Q}^T \end{aligned} \qquad (3\text{-}91)$$

where

$$\overline{Q} = \begin{bmatrix} 1+\Delta & \omega & -\psi \\ -\omega & 1+\Delta & \varepsilon \\ \psi & -\varepsilon & 1+\Delta \end{bmatrix} \qquad (3\text{-}92)$$

3.2.2 Model 3 (Veis) vs. Model 2 (M-Badekas)

Recalling the equation of Model 3 (3-25)

$$\vec{X} = (1+\Delta_V)\ R_V(\vec{U} - \vec{U}_0) + \vec{U}_0 + \vec{T}_V$$

with

$$R_V = R_3^T(\lambda_o)\ R_2^T(90 - \phi_o)\ R_V'\ R_2(90 - \phi_o)\ R_3(\lambda_o)$$

The only difference between Model 2 and Model 3 are the directions of the axes about which the rotations take place.

If one applies the elimination procedure for the rotational parameters as is described in sections 4 and 4.3, one would obtain an identical set of equations for both Model 2 and Model 3. See equations (4-29).

This implies immediately that identical scale and translation parameters will be obtained using either Veis' or M-Badekas' model.

The relationship between the rotational parameters $R_M(\varepsilon, \psi, \omega)$ and $R_V(\phi_o, \lambda_o, \alpha, \xi, \eta)$ are described in section 2.3 (2-22, 2-23 and 2-24).

This similarity between the two models means that the discussions of section 3.2.1 concerning the disadvantages of the M-Badekas model apply to the Veis model as well.

3.3 Comparison of Four-Parameter Transformation Models

Sometimes partial transformation models are used to transform two sets of coordinates. This is done usually when only few stations are available.

In the four-parameter similarity transformation model, it is assumed that the rotational parameters can be neglected.

3.3.1 Model 1 (Bursa) vs. Model 2 (M-Badekas)

The first two models with four transformation parameters (translation and scale) are obtained by taking $R_B = R_M = R = I$ in the models B_7 (3-21) and M_7 (3-23):

$$B_4: \quad \vec{X} = (1 + \Delta_B)\,\vec{U} + \vec{T}_B \tag{3-93}$$

$$M_4: \quad \vec{X} = (1 + \Delta_M)\,(\vec{U} - \vec{U}_o) + \vec{U}_o + \vec{T}_M \tag{3-94}$$

It should be noted that by equating $R_V = I$ in (3-25), the only difference between models M_7 and V_7 has disappeared.

Following the same reasoning as in section 3.2 for the seven-parameter models, one arrives at the conclusions, i.e., both models give the same scale factor, residuals, etc. The following relationship between the translation parameters can be established:

$$\vec{T}_B = \vec{T}_M + \vec{U}_o - (1 + \Delta)\,\vec{U}_o \tag{3-95}$$

Once again, only $\vec{T}_B$ has geometric meaning, appealing to geodetic intuition.

3.3.2 Four-Parameter Models vs. Seven-Parameter Models

Comparing the four-parameter Model 1 (B_4) to the seven-parameter Model 1 (B_7), care should be exercised since the residuals will not generally be equal,

$$B_7: \quad \vec{X}^a_{B_7} = (1 + \Delta_{B_7})\, R\, \vec{U}^a_{B_7} + \vec{T}_{B_7} \tag{3-96}$$

$$B_4: \quad \vec{X}^a_{B_4} = (1 + \Delta_{B_4})\, \vec{U}^a_{B_4} + \vec{T}_{B_4} \tag{3-97}$$

or in terms of observations and residuals,

$$B_7: \quad \vec{X}^b + \vec{V}_{X,B_7} = (1 + \Delta_{B_7})\, R\, (\vec{U}^b + \vec{V}_{U,B_7}) + \vec{T}_{B_7} \tag{3-98}$$

$$B_4: \quad \vec{X}^b + \vec{V}_{X,B_4} = (1 + \Delta_{B_4})(\vec{U}^b + \vec{V}_{U,B_4}) + \vec{T}_{B_4} \tag{3-99}$$

Inspecting the design matrices A'_B and A'_M (3-45) of both Models 1 and 2, one sees that the column pertaining to the scale factor is orthogonal to each of the three columns pertaining to the rotation angles. This orthogonality implies that the scale factors will be equal in both the seven- and four-parameter models:

$$\Delta_{B_7} = \Delta_{M_7} = \Delta_{B_4} = \Delta_{M_4} \tag{3-100}$$

With (3-100) in mind, (3-98) and (3-99) become

$$\vec{T}_{B_7} = \vec{X}^b + \vec{V}_{X,B_7} - (1 + \Delta)\, R\, (\vec{U}^b + \vec{V}_{U,B_7}) \tag{3-101}$$

$$\vec{T}_{B_4} = \vec{X}^b + \vec{V}_{X,B_4} - (1 + \Delta)(\vec{U}^b + \vec{V}_{U,B_4}) \tag{3-102}$$

Subtracting (3-102) from (3-101)

$$\vec{T}_{B_7} - \vec{T}_{B_4} = -(1 + \Delta)\{(R\, \vec{U}^b + R\, \vec{V}_{U,B_7}) - (\vec{U}^b + \vec{V}_{U,B_4})\} + \vec{V}_{X,B_7} - \vec{V}_{X,B_4} \tag{3-103}$$

or

$$\vec{T}_{B_7} = \vec{T}_{B_4} - (1 + \Delta)\{R(\vec{U}^b + \vec{V}_{U,B_7}) - (\vec{U}^b + \vec{V}_{U,B_4})\} + \vec{V}_{X,B_7} - \vec{V}_{X,B_4} \tag{3-104}$$

In case of small rotations ($R \simeq I$) and the assumption, $R \equiv I$, in the four-parameter model does not cause increased V_{U,B_4}'s and V_{X,B_4}'s (but are in the same residual noise as the $\vec{V}_{U,B_7}$'s and $\vec{V}_{X,B_7}$'s), equation (3-104) reveals that the difference between $\vec{T}_{B_7}$ and $\vec{T}_{B_4}$ can hardly be larger than the sum of maximum residual noise of $(\vec{V}_{U,B_7} - \vec{V}_{U,B_4})$ and $(\vec{V}_{X,B_7} - \vec{V}_{X,B_4})$. In other words, the tendency is that the translation parameters in the four-parameter solution of Model 1 (Bursa) will be very similar to those in the seven-parameter solution of Model 1. The similarity between $\vec{T}_{B_7}$ and $\vec{T}_{B_4}$ is illustrated in Table 5-1, section 5.6.

In a similar way, equation (3-104) can be derived comparing the four-parameter Model 2 (M_4) to the seven-parameter Model 2 (M_7):

$$\vec{T}_{M_7} = \vec{T}_{M_4} - (1 + \Delta)\{R_M(\vec{U}^b + \vec{V}_{U,M_7} - \vec{U}_o) - (\vec{U}^b + \vec{V}_{U,M_4} - \vec{U}_o)\} + \vec{V}_{X,M_7} - \vec{V}_{X,M_4} \tag{3-105}$$

For identical reasons the tendency is that the translation parameters from the four-parameter solution of Model 2 (M-Badekas) will be similar to those from the seven-parameter solution of Model 2.

3.4 Comparison of Three-Parameter Transformation Models

This model, which is the simplest, allows only three translation parameters between two sets of coordinates (other three-parameter models, e.g., three rotations only are hardly used in geodesy and therefore are not discussed here).

3.4.1 Model 1 (Bursa) vs. Model 2 (M-Badekas)

The first two models with three transformation parameters (translation only) are obtained by taking $R_B = R_M = R = I$ and $\Delta_B = \Delta_M = \Delta = 0$ in the models B_7 (3-21) and M_7 (3-23):

$$B_3: \qquad \vec{X} = \vec{U} + \vec{T}_{B_3} \tag{3-106}$$

$$M_3: \qquad \vec{X} = \vec{U} + \vec{T}_{M_3} \tag{3-107}$$

Clearly, both models are identical.

3.4.2 Three-Parameter Models vs. Seven-Parameter Models

For similar reasons as in section 3.3.2, the residuals have to be considered in comparing, e.g., the three-parameter Model 1 (B_3) to the seven-parameter Model 1 (B_7),

$$B_7: \qquad \vec{X}^b + \vec{V}_{X,B_7} = (1 + \Delta)\ R\ (\vec{U}^b + \vec{V}_{U,B_7}) + \vec{T}_{B_7} \tag{3-108}$$

$$B_3: \qquad \vec{X}^b + \vec{V}_{X,B_3} = \vec{U}^b + \vec{V}_{U,B_3} + \vec{T}_{B_3} \tag{3-109}$$

Solving for the translation parameters and subsequent subtracting yields

$$\vec{T}_{B_7} - \vec{T}_{B_3} = -(1 + \Delta)R(\vec{U}^b + \vec{V}_{U,B_7}) + (\vec{U}^b + \vec{V}_{U,B_3}) + \vec{V}_{X,B_7} - \vec{V}_{X,B_3} \tag{3-110}$$

or

$$\vec{T}_{B_7} = \vec{T}_{B_3} - (1 + \Delta)R(\vec{U}^b + \vec{V}_{U,B_7}) + (\vec{U}^b + V_{U,B_3}) + \vec{V}_{X,B_7} - \vec{V}_{X,B_3} \tag{3-111}$$

Recalling the comparison of the full Models 1 (B_7) and (M_7), and especially equation (3-35):

$$\vec{T}_{B_7} = \vec{T}_{M_7} - (1 + \Delta)R(\vec{U}_o) + \vec{U}_o \qquad (3\text{-}35)$$

the resemblance between (3-35) and (3-111) is striking, realizing that in a local area (geodetic datum)

$$\vec{U}_o \approx \vec{U}^b$$

With the assumption that $R \equiv I$, and $\Delta = 0$ in the three-parameter model does not cause increased V_{U,B_3}'s and V_{X,B_3}'s (but are in the same residual noise as the V_{U,B_7}'s and V_{X,B_7}'s), equations (3-111) and (3-35) reveal that $\vec{T}_{B_3}$ will generally be close to $\vec{T}_{M_7}$. In other words, the tendency is that the translation parameters in the three-parameter solution of Model 1 (Bursa) will be very similar to those in the seven-parameter solution of Model 2 (M-Badekas). See Table 5-1, section 5-6. The main conclusion is that three-parameter transformations should not be used if Δ is significant.

This agreement between the translation parameters of M_7 and B_3 might have been the reason that Badekas' interpretation of Molodenskii's formulas enjoyed such an interest by geodesists for such a long time.

3.5 Similarity Transformation Models for More Than Two Sets of Coordinates

Recently models have been proposed to connect more than two sets of coordinates by similarity transformations. One such proposed model, described in section 2.4, transforms a satellite system on one hand and several geodetic datums on the other hand into the Average Terrestrial system. The "component-transformations" can be any variation of Models 1 (Bursa), 2 (M-Badekas) or 3 (Veis).

The quoted Model 4 (Vanicek-Wells) consists of a six-parameter Model 1 (a satellite to Average Terrestrial) on one hand and a five-parameter Model 1 (geodetic to Average Terrestrial) on the other hand (equation 3-26).

The only remark which can be made regarding the geometrical interpretation of the translation vector $\vec{T}_{SG}$ (= $\vec{T}_G - \vec{T}_S$, see equations 2-25 and 2-26), denoting the translation of the geodetic datum with respect to the satellite system, is that its components are expressed neither in the geodetic nor in the satellite system, but in the Average Terrestrial system.

3.6 Conclusions

(1) Only Model 1 (Bursa, seven parameter) gives a geometrically meaningful translation vector between the origin of both coordinate systems. The translations as obtained from Model 1 and Model 2 are fully interdependent and their relation is expressed by equation (3-35).

(2) Both sets of parameters as obtained from Model 1 (Bursa) and Model 2 (M-Badekas) are completely equivalent when used in direct transformations, i.e., the same coordinates and variance-covariance matrices are obtained for the transformed coordinates.

(3) The correlation coefficients between parameters are significantly higher in Model 1 (Bursa) than in Model 2 (M-Badekas). The correlation coefficients are a function of the selected mathematical model. They reflect the geometry with respect to that particular model. It should be recognized that the decreasing magnitude of the correlation coefficients in Model 2 is essentially due to the different definition of the transformation parameters.

(4) The translation vector as obtained from the seven-parameter Model 2 (M-Badekas) is generally close to the translation vector as obtained from the three-parameter Model 1 (Bursa). For this reason three-parameter transformations should be used only when $\Delta \approx 0$.

(5) Model 2 (M-Badekas) and Model 3 (Veis) are identical except for a redefinition of the axes about which the rotations take place. They give identical scale factor and translation components.

(6) Model 2 was attributed to Molodenskii by Badekas (1969) presumably based on the treatment in (Molodenskii et al., 1962, page 14). At this point, however, it should be questioned whether the transition

from Molodenskii's derivation (Molodenskii et al., 1962) to Badekas' so-called "Molodenskii Model" (Badekas, 1969) can at all be done or has been done correctly (Soler, 1976).

(7) If a four-parameter transformation is performed (translation and scale) both Model 1 and Model 2 give the same scale factor which is identical to the scale factor obtained in a seven-parameter solution.

4. SOME PITFALLS TO BE AVOIDED IN SIMILARITY TRANSFORMATIONS

Methods to compute similarity transformation parameters separately have been proposed in the geodetic literature as early as 1967 (Bursa, 1967). In this reference, a method is described to determine the rotational parameters from directions. A similar approach to compute the scale factor also separately from chord comparisons can be found in (Kumar, 1972). In the latter reference, this line of thought is extended to the estimation procedure: from separate adjustments some similarity transformation parameters are computed together with their variances/covariances. In the subsequent adjustment of the general similarity transformation model, these separately computed parameters are weight-constrained using the earlier obtained (co-)variances. The selection of coordinate functions (directions, chords) in (Bursa, 1967) as well as in (Kumar, 1972) is such that it contains all combinations between the points (e.g., in case of n points all $\frac{1}{2}n(n-1)$ possible chords are used). In this section it will be shown that:

<u>A</u>. If two sets of points differ by a similarity transformation, the chords between points do not yield additional information concerning scale, directions do not add information concerning rotation, and chord ratios do not yield extra information concerning translation, contrary to (Kumar, 1972).

<u>B</u>. The three coordinate functions (chords, directions and chord ratios) only furnish methods to compute similarity transformation parameters separately. However, great care should be exercised in selecting the set of coordinate functions as overlooked in (Bursa, 1967 and Kumar, 1972).

In the following discussion, the A1 model (3-15) is used as an example.

$$\vec{X} = \lambda R \vec{U} + \vec{T} \qquad (4\text{-}1)$$

This model was previously referred to as the Bursa model.

Similarity Transformation: General Adjustment or Stepwise Adjustment?

If a set of coordinates of n points is given in two different coordinate systems, it is often desired to know the set of seven parameters which transforms one system into the other (assuming that the hypothesis of similarity between the two systems is justified).

A set of n points yields 3n equations with seven unknowns,

$$\vec{X}_i = \lambda R \vec{U}_i + \vec{T} \qquad i = 1, \ldots, n \tag{4-2}$$

or

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \lambda \begin{bmatrix} \cos\psi \cos\omega & \cos\varepsilon \sin\omega + \sin\varepsilon \sin\psi \cos\omega & \sin\varepsilon \sin\omega - \cos\varepsilon \sin\psi \cos\omega \\ -\cos\psi \sin\omega & \cos\varepsilon \cos\omega - \sin\varepsilon \sin\psi \sin\omega & \sin\varepsilon \cos\omega + \cos\varepsilon \sin\psi \sin\omega \\ \sin\psi & -\sin\varepsilon \cos\psi & \cos\varepsilon \cos\psi \end{bmatrix} \times$$

$$\times \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \tag{4-3}$$

Clearly, at this stage a least squares adjustment could be performed on this set of (nonlinear) equations with degrees of freedom $3n - 7$.

4.1 Scale from Chords

4.1.1 Mathematical Derivation

From equation (4-3) it is readily recognized that before performing a least squares adjustment one might eliminate three parameters (x_0, y_0, z_0) by subtracting, for instance, the set of equations for point 1 ($i = 1$) from all the other following equations:

$$\begin{bmatrix} x_i - x_1 \\ y_i - y_1 \\ z_i - z_1 \end{bmatrix} = \lambda \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} u_i - u_1 \\ v_i - v_1 \\ w_i - w_1 \end{bmatrix} \qquad i = 2, \ldots, n \tag{4-4}$$

Equation (4-4) is identical if the original equations are either

$$\vec{X} = \lambda R \vec{U} + \vec{T} \qquad \text{Bursa}$$

or

$$\vec{X} = \lambda R(\vec{U} - \vec{U}_o) + \vec{U}_o + \vec{T} \qquad \text{M-Badekas}$$

Thus, the scale and rotational parameters in both models are identical.

At this point a least squares adjustment of these coordinate differences would yield a solution vector for the parameters ε, ψ, ω and λ. Set (4-4) is a set of 3(n-1) equations with four unknowns. Still, the degrees of freedom are 3(n-1) - 4 = 3n - 7.

Set (4-4) written out gives

$$(x_i - x_1) = \lambda \{R_{11}(u_i - u_1) + R_{12}(v_i - v_1) + R_{13}(w_i - w_1)\} \tag{4-5}$$

$$(y_i - y_1) = \lambda \{R_{21}(u_i - u_1) + R_{22}(v_i - v_1) + R_{23}(w_i - w_1)\} \quad i = 2, \ldots, n \tag{4-6}$$

$$(z_i - z_1) = \lambda \{R_{31}(u_i - u_1) + R_{32}(v_i - v_1) + R_{33}(w_i - w_1)\} \tag{4-7}$$

Now eliminate the three rotation angles ε, ψ and ω from the above equations. Introducing the following notation:

$$\begin{aligned} x_{i1} &= x_i - x_1 \\ y_{i1} &= y_i - y_1 \\ &\text{etc.} \end{aligned} \tag{4-8}$$

and squaring equations (4-5), (4-6) and (4-7) gives

$$\begin{aligned} \Delta x_{i1}^2 = \lambda^2(&R_{11}^2 \Delta u_{i1}^2 + R_{12}^2 \Delta v_{i1}^2 + R_{13}^2 \Delta w_{i1}^2 + 2R_{11}R_{12}\Delta u_{i1}\Delta v_{i1} + \\ &+ 2R_{11}R_{13}\Delta u_{i1}\Delta w_{i1} + 2R_{12}R_{13}\Delta v_{i1}\Delta w_{i1}) \end{aligned} \tag{4-9}$$

$$\Delta y_{i1}^2 = \lambda^2(R_{21}^2\Delta u_{i1}^2 + R_{22}^2\Delta v_{i1}^2 + R_{23}^2\Delta w_{i1}^2 + 2R_{21}R_{22}\Delta u_{i1}\Delta v_{i1} + 2R_{21}R_{23}\Delta u_{i1}\Delta w_{i1} + 2R_{22}R_{23}\Delta v_{i1}\Delta w_{i1}) \qquad (4\text{-}10)$$

$$\Delta z_{i1}^2 = \lambda^2(R_{31}^2\Delta u_{i1}^2 + R_{32}^2\Delta v_{i1}^2 + R_{33}^2\Delta w_{i1}^2 + 2R_{31}R_{32}\Delta u_{i1}\Delta v_{i1} + 2R_{31}R_{33}\Delta u_{i1}\Delta w_{i1} + 2R_{32}R_{33}\Delta v_{i1}\Delta w_{i1}) \qquad (4\text{-}11)$$

$$i = 2, \ldots, n$$

The elimination of the three unknowns ε, ψ, ω from these 3(n-1) nonlinear equations is not obvious and great care should be exercised.

Looking at equations (4-9), (4-10) and (4-11) and keeping in mind the conditions imposed on R_{ij} (see (3-4)), it may be seen that adding each set of three equations for each point i results in

$$\Delta x_{i1}^2 + \Delta y_{i1}^2 + \Delta z_{i1}^2 = \lambda^2(\Delta u_{i1}^2 + \Delta v_{i1}^2 + \Delta w_{i1}^2) \qquad i = 2, \ldots, n \quad (4\text{-}12)$$

This is a set of (n-1) equations with one unknown, which means (n-2) degrees of freedom.

Going from (4-9) - (4-11) to (4-12), (3n-7) - (n-2) = (2n-5) degrees of freedom have been lost, which is obviously wrong.

One has to go back to the set (4-5) - (4-7) to see what actually happened. If each equation in that set is numbered and represented symbolically by [i], one sees that equations (4-5) - (4-7), looking like

[1]
[2]
[3]
[4]
⋮
[3n-3],

have been transformed to a set of equations (4-12) looking like

$$[1]^2 + [2]^2 + [3]^2$$

$$[4]^2 + [5]^2 + [6]^2$$

$$\vdots$$

$$[3n-5]^2 + [3n-4]^2 + [3n-3]^2$$

This implies that the three rotational parameters are not eliminated from the combinations of coordinate differences, which are not dependent on each point. In other words combinations consisting of [1], [4], [5] or [2], [4], [5] etc. are omitted.

This omission can be accounted for by subtracting the set of coordinate differences (4-5) - (4-7) for $i = 2$ from all the other coordinate differences, yielding

$$\Delta x_{j2} = \lambda(R_{11}\Delta u_{j2} + R_{12}\Delta v_{j2} + R_{13}\Delta w_{j2}) \tag{4-13}$$

$$\Delta y_{j2} = \lambda(R_{21}\Delta u_{j2} + R_{22}\Delta v_{j2} + R_{23}\Delta w_{j2}) \qquad j = 3, \ldots, n \tag{4-14}$$

$$\Delta z_{j2} = \lambda(R_{31}\Delta u_{j2} + R_{32}\Delta v_{j2} + R_{33}\Delta w_{j2}) \tag{4-15}$$

Following the same procedure of squaring and adding, the result becomes

$$\Delta x_{j2}^2 + \Delta y_{j2}^2 + \Delta z_{j2}^2 = \lambda^2(\Delta u_{j2}^2 + \Delta y_{j2}^2 + \Delta w_{j2}^2) \qquad j = 3, \ldots, n \tag{4-16}$$

This is a set of (n-2) equations with one unknown. The same procedure applied to the coordinate differences (4-13) - (4-15) by subtracting the first set (4-13) - (4-15) for $j = 3$ from all the other coordinate differences, yields

$$\Delta x_{k3} = \lambda(R_{11}\Delta u_{k3} + R_{12}\Delta y_{k3} + R_{13}\Delta w_{k3}) \tag{4-17}$$

$$\Delta y_{k3} = \lambda(R_{21}\Delta u_{k3} + R_{22}\Delta v_{k3} + R_{23}\Delta w_{k3}) \qquad k = 4, \ldots, n \tag{4-18}$$

$$\Delta z_{k3} = \lambda(R_{31}\Delta u_{k3} + R_{32}\Delta v_{k3} + R_{33}\Delta w_{k3}) \tag{4-19}$$

Again, squaring and adding results in

$$\Delta x_{k3}^2 + \Delta y_{k3}^2 + \Delta z_{k3}^2 = \lambda^2(\Delta u_{k3}^2 + \Delta v_{k3}^2 + \Delta w_{k3}^2) \quad k = 4, \ldots, n \tag{4-20}$$

Collecting the sets (4-12), (4-16) and (4-20),

$$\begin{aligned}
\Delta x_{i1}^2 + \Delta y_{i1}^2 + \Delta z_{i1}^2 &= \lambda^2(\Delta u_{i1}^2 + \Delta v_{i1}^2 + \Delta w_{i1}^2) \quad i=2, \ldots, n \\
\Delta x_{j2}^2 + \Delta y_{j2}^2 + \Delta z_{j2}^2 &= \lambda^2(\Delta u_{j2}^2 + \Delta v_{j2}^2 + \Delta w_{j2}^2) \quad j=3, \ldots, n \\
\Delta x_{k3}^2 + \Delta y_{k3}^2 + \Delta z_{k3}^2 &= \lambda^2(\Delta u_{k3}^2 + \Delta v_{k3}^2 + \Delta w_{k3}^2) \quad k=4, \ldots, n
\end{aligned} \tag{4-21}$$

This is a set of (n-1) + (n-2) + (n-3) = (3n-6) equations with one unknown. Now the degrees of freedom of (3n-7) has been preserved. Symbolically, (4-21) can be written as

$$\left\{\begin{array}{l}
[1]^2 + [2]^2 + [3]^2 \\
[4]^2 + [5]^2 + [6]^2 \\
\qquad\vdots \\
[3n-5]^2 + [3n-4]^2 + [3n-3]^2 \quad \text{which are (n-1) equations}
\end{array}\right.$$

$$\left\{\begin{array}{l}
([4]-[1])^2 + ([5]-[2])^2 + ([6]-[3])^2 \\
([7]-[1])^2 + ([8]-[2])^2 + ([9]-[3])^2 \\
\qquad\vdots \\
([3n-5]-[1])^2 + ([3n-4]-[2])^2 + ([3n-3]-[3])^2 \quad \text{which are (n-2) equations}
\end{array}\right.$$

$$\left\{\begin{array}{l}
([7]-[4])^2 + ([8]-[5])^2 + ([9]-[6])^2 \\
([10]-[4])^2 + ([11]-[5])^2 + ([12]-[6])^2 \\
\qquad\vdots \\
([3n-5]-[4])^2 + ([3n-4]-[5])^2 + ([3n-3]-[6])^2 \quad \text{which are (n-3) equations}
\end{array}\right.$$

the chord distances between the points 1, 2 and 3 and any other point of the complete set of points. Consequently, (4-21) can be written as

$$R_{X_{ij}} = \lambda \; R_{U_{ij}} \qquad \begin{array}{l} i = 1, 2, 3 \\ j = 1, \ldots, n \\ j > 1 \end{array} \tag{4-22}$$

It has been shown that computing a scale factor λ from the general similarity transformation model (4-2) or (4-3) is equivalent to the computation of the scale factor λ doing a comparison of a proper set of chords (4-22). A numerical example shows that the outcome of the scale factor and its variance are identical using both methods (see section 4.5).

4.1.2 Geometrical Verification

The derivation in the previous section can be verified geometrically. If one thinks of positions of points determined by their distances to other points, then each point is determined in position by three distances to three other points (Fig. 4-1).

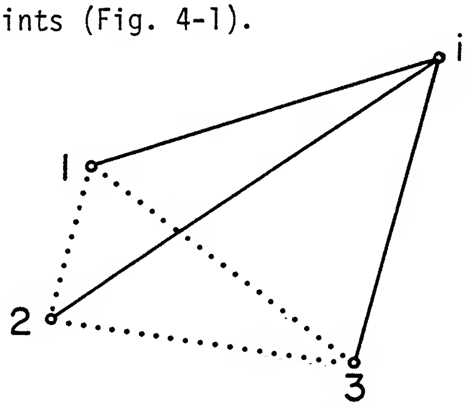


Fig. 4-1. Positioning by Chords

If one has a set of five points and conveniently determines each point in position with respect to the same three points, the following set of chords are obtained (Fig. 4-2):

$$\begin{array}{lll} r_{12} & r_{23} & r_{34} \\ r_{13} & r_{24} & r_{35} \\ r_{14} & r_{25} & \\ r_{15} & & \end{array}$$

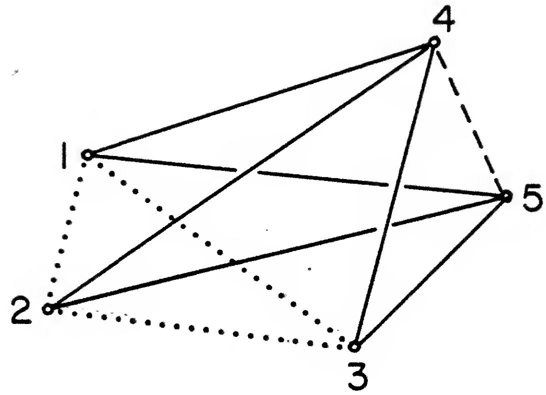


Fig. 4-2. Positioning by Chords (5 Points)

The above set of chords forms a geometrically independent set of chords in the sense that the chord r_{45} can be omitted as a tenth chord since r_{45} can be computed from the other nine (independent) chords.

Consequently, in an adjustment procedure where the scale factor is going to be computed from chords (4-22) between n points, only an independent set of chords should be used and not the set of all possible combinations of chords. An independent set of chords between n points contains

$$(n-1) + (n-2) + (n-3) = 3n-6 \text{ chords}$$

while the set of chords in all possible combinations contains

$$n(n-1)/2 \text{ chords!}$$

Every attempt to obtain a least squares solution using (4-22) with all $n(n-1)/2$ combinations will fail due to singularity problems.

Denoting (4-22) as in (Uotila, 1967) by

$$F(L_a, X_a) = F(L_b + V, X_o + X) = 0$$

where

L_b are the observations (in this case the chords in both coordinate systems)

X_a are the unknowns (in this case the scale factor λ)($X_a = X_o + X$, X obtained from least squares adjustment)

with $A = \partial F/\partial X_a$, $B = \partial F/\partial L_a$, $W = F(L_b, X_o)$ and $\Sigma L_b = \sigma_o^2 P^{-1}$ (variance/covariance matrix of the observations), one obtains

$$X_a = X_o - (A^T M^{-1} A)^{-1} A^T M^{-1} W \qquad (= \lambda_{adjusted})$$

and
$$M = B P^{-1} B^T$$

If a set of chords in all possible combinations is being used, the matrix M is singular due to the dependency in the observations. The procedure as mentioned in (Kumar, 1972) that the scale factor can be obtained from n(n-1)/2 chords is therefore in error.

The reason that the singularity of matrix M was never felt is due to the fact that the scale factor λ was not obtained through a rigorous least squares adjustment of (4-22) but was obtained in a weighted mean sense. Basically, this method neglects the off-diagonal elements in the matrix M so that this matrix suddenly becomes invertable.

As one can see from section 4.5, the scale factor λ as obtained from the weighted mean method will not differ too much from the scale factor as obtained from the rigorous least squares adjustment. However, due to the fact that

$$n(n-1)/2 \gg 3n-6$$

for a large n the difference between the degrees of freedom may yield an unrealistically small variance for the scale factor if the weighted mean procedure is used.

An apparent disadvantage of the chord comparison is that the property of having each point equally represented in number in the model (4-3) is lost. Moreover, a number of independent sets of chords can be generated;

for instance, the method of determining each point (4, ..., n) with respect to a basic set of points (1, 2, 3) was mentioned earlier. This basic set can be changed in a number of ways, but always results in overrepresenting those points in the equations which form that basic set, e.g.,

$$
\begin{array}{ccc}
r_{12} & r_{23} & r_{34} \\
r_{13} & r_{24} & r_{35} \\
r_{14} & r_{25} & r_{36} \\
r_{15} & r_{26} & r_{37} \\
r_{16} & r_{27} & \\
r_{17} & &
\end{array}
$$

A method which approaches closest the equal representation is the one in which each point is determined in position with respect to its three preceding points, e.g.:

$$
\begin{array}{ccccc}
r_{12} & r_{14} & r_{25} & r_{36} & r_{47} \\
r_{13} & r_{24} & r_{35} & r_{46} & r_{57} \\
r_{23} & r_{34} & r_{45} & r_{56} & r_{67}
\end{array}
$$

A numerical example shows that the outcome of the scale factor is not dependent on the choice of the basic set (section 4.5).

4.2 Orientation from Directions

4.2.1 Mathematical Derivation

As in section 4.1.1 for the scale factor, it is possible to compute the orientation angles exclusively from two sets of coordinates by eliminating parameters from equation (4-3) in a proper sequence.

First of all, the origin parameters are eliminated from the equations by subtraction. (See equations (4-4) and (4-5) - (4-7).) Instead of eliminating the orientation angles to obtain the scale factor exclusively, now the scale factor will be eliminated first in order to be left with equations in which the orientation angles are the only parameters.

Starting from equations (4-5) - (4-7), the scale factor can be eliminated by dividing the second equation by the first, and the third equation by the square root of the sum of the first two for each point i. Thus

$$\frac{\Delta y_{i1}}{\Delta x_{i1}} = \frac{R_{21}\Delta u_{i1} + R_{22}\Delta v_{i1} + R_{23}\Delta w_{i1}}{R_{11}\Delta u_{i1} + R_{12}\Delta v_{i1} + R_{13}\Delta w_{i1}} \quad . \qquad (4\text{-}23)$$

$$\frac{\Delta z_{i1}}{\sqrt{\Delta x_{i1}^2 + \Delta y_{i1}^2}} = \frac{R_{31}\Delta u_{i1} + R_{32}\Delta v_{i1} + R_{33}\Delta w_{i1}}{\sqrt{DN}} \qquad i = 2, \ldots, n \quad (4\text{-}24)$$

with

$$\begin{aligned} DN = {} & (R_{11}^2 + R_{21}^2)\Delta u_{i1}^2 + (R_{12}^2 + R_{22}^2)\Delta v_{i1}^2 + (R_{13}^2 + R_{23}^2)\Delta w_{i1}^2 + \\ & + 2(R_{11}R_{12} + R_{21}R_{22})\Delta u_{i1}\Delta v_{i1} + 2(R_{11}R_{13} + R_{21}R_{23})\Delta u_{i1}\Delta w_{i1} + \\ & + 2(R_{12}R_{13} + R_{22}R_{23})\Delta v_{i1}\Delta w_{i1} \end{aligned} \qquad (4\text{-}25)$$

This set (4-23) - (4-25) contains 2(n-1) equations with three unknowns (ε, ψ, ω) which yields 2n-5 degrees of freedom which obviously is not sufficient to obtain the required 3n-7 degrees of freedom. The cause is that the scale factor λ is only eliminated in each triplet of equations (three equations per point).

In a similar way as in section 4.1.1 in equations (4-13) - (4-15), the complementary set of (independent) equations are obtained by using the coordinate differences as computed for point 2.

From equations (4-13) - (4-15) one gets directly

$$\frac{\Delta y_{j2}}{\Delta x_{j2}} = \frac{R_{21}\Delta u_{j2} + R_{22}\Delta v_{j2} + R_{23}\Delta w_{j2}}{R_{11}\Delta u_{j2} + R_{12}\Delta v_{j2} + R_{13}\Delta w_{j2}} \qquad j = 3, \ldots, n \tag{4-26}$$

This set gives (n-2) additional independent equations. The correct degrees of freedom are then 2(n-1) + (n-2) - 3 = 3n-7. Collecting the sets of equations (4-23), (4-24) and (4-26), one has

$$\frac{\Delta y_{i1}}{\Delta x_{i1}} = \frac{R_{21}\Delta u_{i1} + R_{22}\Delta v_{i1} + R_{23}\Delta w_{i1}}{R_{11}\Delta u_{i1} + R_{12}\Delta v_{i1} + R_{13}\Delta w_{i1}}$$

$$\frac{\Delta z_{i1}}{\sqrt{\Delta x_{i1}^2 + \Delta y_{i1}^2}} = \frac{R_{31}\Delta u_{i1} + R_{32}\Delta v_{i1} + R_{33}\Delta w_{i1}}{\sqrt{DN}} \qquad \begin{array}{l} i = 2, \ldots, n \\ j = 3, \ldots, n \end{array} \tag{4-27}$$

$$\frac{\Delta y_{j2}}{\Delta x_{j2}} = \frac{R_{21}\Delta u_{j2} + R_{22}\Delta v_{j2} + R_{23}\Delta w_{j2}}{R_{11}\Delta u_{j2} + R_{12}\Delta v_{j2} + R_{13}\Delta w_{j2}}$$

$$\begin{aligned} DN = {} & (R_{11}^2 + R_{21}^2)\Delta u_{i1}^2 + (R_{12}^2 + R_{22}^2)\Delta v_{i1}^2 + (R_{13}^2 + R_{23}^2)\Delta w_{i1}^2 + \\ & + 2(R_{11}R_{12} + R_{21}R_{22})\Delta u_{i1}\Delta v_{i1} + 2(R_{11}R_{13} + R_{21}R_{23})\Delta u_{i1}\Delta w_{i1} + \\ & + 2(R_{12}R_{13} + R_{22}R_{23})\Delta v_{i1}\Delta w_{i1} \end{aligned}$$

Notice that the set of equations (4-27) represents two topocentric angles α and δ defining directions in local systems parallel to the XYZ reference system (Fig. 4-3).

$$\tan \alpha_{i1} = \frac{\Delta y_{i1}}{\Delta x_{i1}}$$

$$\tan \delta_{i1} = \frac{\Delta z_{i1}}{\sqrt{\Delta x_{i1}^2 + \Delta y_{i1}^2}} \qquad \begin{array}{l} i = 2, \ldots, n \\ j = 3, \ldots, n \end{array} \tag{4-28}$$

$$\tan \alpha_{j2} = \frac{\Delta y_{j2}}{\Delta x_{j2}}$$

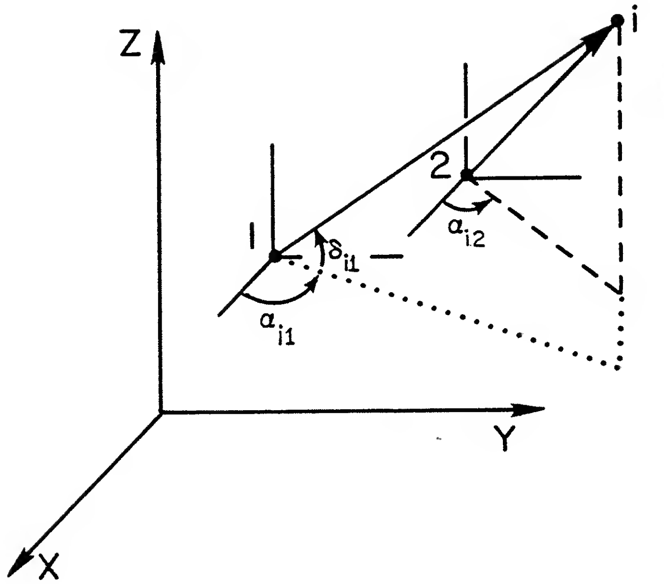


Fig. 4-3. Topocentric Angles α and δ

It has been shown that computing the orientation angles ε, ψ, ω from the general similarity model (4-2) or (4-3) is equivalent to the computation of the orientation angles ε, ψ, ω using a proper set of directions.

4.2.2 Geometrical Verification

If one is allowed to determine the relative positions of n points only with the help of directions, the following observations can be made.

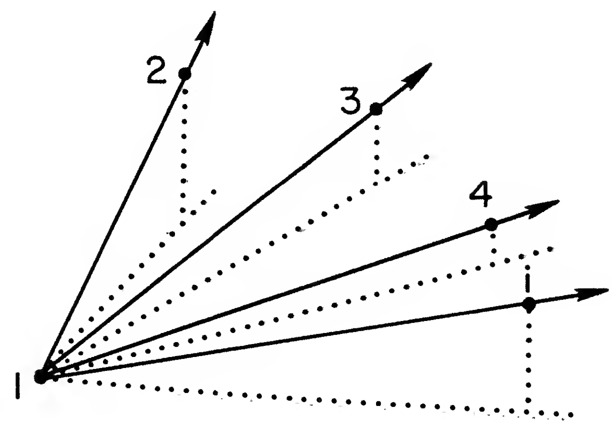


Fig. 4-4. Positioning by Directions (from One Point)

Having a set of directions originating at one point (Fig. 4-4) leaves the other points only one degree of freedom in relative position with respect to point 1 (i.e., the freedom to move along line 1i). This initial set contains then 2(n-1) directional angles (α, δ).

The last degree of freedom can easily be resolved, for example, by giving directions from point 2 to all other points. Since the directional vector can only move in the plane spanned by the directional vectors 12 and 1i, only one quantity (α or δ) is needed to determine the relative position of point j with respect to points 1 and 2 (Fig. 4-5).

Consequently, an independent set of directions between n points contains

$$2(n-1) + (n-2) = 3n-4 \text{ directional angles}$$

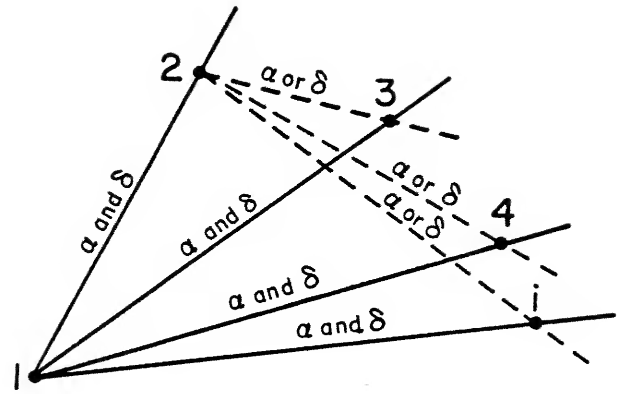


Fig. 4-5. Positioning by Directions (from Two Points)

The procedure as mentioned in (Bursa, 1967) that the rotating elements between two coordinate systems can be obtained from all possible n(n-1) directional angles is in error for the same reasons as extensively explained in section 4.1.2.

In a similar way, unrealistically small variances will be obtained for the orientation angles if the weighted mean method is used (Bursa, 1967, p. 392). Once more, if a strict procedure of propagation of error is followed, least squares estimates for the orientation angles cannot be obtained due to singularity of a matrix caused by the dependency in the observations.

4.3 Origin from Chord Ratios

A procedure similar to the one with which the scale factor from chords was obtained can be followed to compute the origin from chord ratios. One recalls that the procedure of eliminating parameters as used in section 4.1.1 resulted in chords generated from three points to all other points. If chords are now generated from the (unknown) origin and two other points, one gets a set of equations almost identical to (4-21),

$$
\begin{aligned}
\Delta x_{io}^2 + \Delta y_{io}^2 + \Delta z_{io}^2 &= \lambda^2(\Delta u_{io}^2 + \Delta v_{io}^2 + \Delta w_{io}^2) \quad i=1, \ldots, n \\
\Delta x_{j1}^2 + \Delta y_{j1}^2 + \Delta z_{j1}^2 &= \lambda^2(\Delta u_{j1}^2 + \Delta v_{j1}^2 + \Delta w_{j1}^2) \quad j=2, \ldots, n \\
\Delta x_{k2}^2 + \Delta y_{k2}^2 + \Delta z_{k2}^2 &= \lambda^2(\Delta u_{k2}^2 + \Delta v_{k2}^2 + \Delta w_{k2}^2) \quad k=3, \ldots, n
\end{aligned} \tag{4-29}
$$

Recognizing that $u_o = v_o = w_o = 0$, we get

$$
\begin{aligned}
\Delta x_{io}^2 + \Delta y_{io}^2 + \Delta z_{io}^2 &= \lambda^2(\Delta u_{i}^2 + \Delta v_{i}^2 + \Delta w_{i}^2) \quad i=1, \ldots, n \\
\Delta x_{j1}^2 + \Delta y_{j1}^2 + \Delta z_{j1}^2 &= \lambda^2(\Delta u_{j1}^2 + \Delta v_{j1}^2 + \Delta w_{j1}^2) \quad j=2, \ldots, n \\
\Delta x_{k2}^2 + \Delta y_{k2}^2 + \Delta z_{k2}^2 &= \lambda^2(\Delta u_{k2}^2 + \Delta v_{k2}^2 + \Delta w_{k2}^2) \quad k=3, \ldots, n
\end{aligned} \tag{4-30}
$$

This set (4-30) consists of n + (n-1) + (n-2) = 3n-3 equations with four unknowns λ, x_o, y_o, z_o. The degrees of freedom are still (3n-3) - 4 = 3n-7. If the scale factor is computed from the first equation for i = 1 and substituted in the remaining equations, one gets

$$
\begin{aligned}
&(\Delta x_{io}^2 + \Delta y_{io}^2 + \Delta z_{io}^2)/(\Delta u_{i}^2 + \Delta v_{i}^2 + \Delta w_{i}^2) = \\
&\qquad (\Delta x_{1o}^2 + \Delta y_{1o}^2 + \Delta z_{1o}^2)/(\Delta u_{1}^2 + \Delta v_{1}^2 + \Delta w_{1}^2) \quad i=2, \ldots, n \\
&(\Delta x_{j1}^2 + \Delta y_{j1}^2 + \Delta z_{j1}^2)/(\Delta u_{j1}^2 + \Delta v_{j1}^2 + \Delta w_{j1}^2) = \\
&\qquad (\Delta x_{1o}^2 + \Delta y_{1o}^2 + \Delta z_{1o}^2)/(\Delta u_{1}^2 + \Delta v_{1}^2 + \Delta w_{1}^2) \quad j=2, \ldots, n \\
&(\Delta x_{k2}^2 + \Delta y_{k2}^2 + \Delta z_{k2}^2)/(\Delta u_{k2}^2 + \Delta v_{k2}^2 + \Delta w_{k2}^2) = \\
&\qquad (\Delta x_{1o}^2 + \Delta y_{1o}^2 + \Delta z_{1o}^2)/(\Delta u_{1}^2 + \Delta v_{1}^2 + \Delta w_{1}^2) \quad k=3, \ldots, n
\end{aligned} \tag{4-31}
$$

Recognizing that the set of equations (4-31) are relationships between ratios of chords squared, (4-31) might be simplified by taking the square root

$$
\begin{aligned}
R_{X_{io}}/R_{U_{io}} &= R_{X_{1o}}/R_{U_{1o}} \quad i=2, \ldots, n \\
R_{X_{j1}}/R_{U_{j1}} &= R_{X_{1o}}/R_{U_{1o}} \quad j=2, \ldots, n \\
R_{X_{k2}}/R_{U_{k2}} &= R_{X_{1o}}/R_{U_{1o}} \quad k=3, \ldots, n
\end{aligned} \tag{4-32}
$$

This set (4-32) consists of (n-1) + (n-1) + (n-2) = 3n-4 equations with three unknowns x_0, y_0, z_0. The degrees of freedom are preserved: (3n-4) - 3 = 3n-7.

Even shorter

$$R_{X_{ij}}/R_{U_{ij}} = R_{X_{1o}}/R_{U_{1o}} \qquad \begin{array}{l} i=0,\ 1,\ 2 \\ j=2,\ \ldots,\ n \\ j > i \end{array} \tag{4-33}$$

It has been shown that computing the origin x_0, y_0, z_0 from the general similarity model (4-2) or (4-3) is equivalent to the computation of the origin x_0, y_0, z_0 doing a comparison of a proper set of chord ratios.

4.4 Summary

Sections 4.1 through 4.3 might be summarized in the following diagram:

General Similarity Transformation Model
Between Coordinate Systems X and U

$$\vec{X} = \lambda R\vec{U} + \vec{T} \quad (\lambda, \varepsilon, \psi, \omega, x_o, y_o, z_o)$$

Parameters to Be Obtained Separately	Scale Factor λ	Orientation $\varepsilon, \psi, \omega$	Origin x_o, y_o, z_o
Parameters to Be Eliminated FIRST from General Model	Origin x_o, y_o, z_o	Origin x_o, y_o, z_o	Orientation $\varepsilon, \psi, \omega$
Resulting Model	Coordinate Differences	Coordinate Differences	Chords
Parameters to Be Eliminated SECONDLY from Previous Model	Orientation $\varepsilon, \psi, \omega$	Scale Factor λ	Scale Factor λ
RESULTING MODEL	CHORDS $R_{X_{ij}} = \lambda R_{U_{ij}}$ (4-22) $i=1, 2, 3$ $j=1, \ldots, n$ $j > i$	DIRECTIONS $\tan \alpha_{i1} = \ldots$ $\tan \delta_{i1} = \ldots$ $\tan \alpha_{j2} = \ldots$ (4-27) $i=2, \ldots, n$ $j=3, \ldots, n$	CHORD RATIOS $R_{X_{ij}}/R_{U_{ij}} =$ $R_{X_{1o}}/R_{U_{1o}}$ (4-33) $i=0, 1, 2$ $j=2, \ldots, n$ $j > i$

This diagram and the previous sections show just one way of obtaining specified sets of parameters separately.

Taking section 4.2.1 as an example, the scale factor λ has been eliminated in such a way ((4-23) and (4-24)) that the geometrical interpretation of "local geodetic" hour angle and declination was arrived at. An easier way of eliminating the scale factor λ is to divide all the coordinate

differences (4-5) - (4-7) by Δx_{i1}. In this case the title of section 4.2.1 could have been: Orientation from Ratios of Coordinate Differences.

The reader might be able to derive even more ways of parameter elimination resulting in final equations which have different geometrical interpretations but will eventually yield the same values.

The final recommendation might be that no attempt should be made to split up the general similarity model into partial models for the following reasons: (a) the results will be identical since the partial models do not contain additional information about transformation parameters; (b) since the results will be identical, all seven transformation parameters will be obtained in one single adjustment. It should be noted that the similarity does not only refer to the value of the transformation parameters but also to their variances and covariances.

4.5 Numerical Example

An example has been computed using 13 stations in Europe. The coordinates were given both in the European Datum 1950 System and in the OSU WN14 System, as derived from satellite observations (Leick et al., 1975).

The scale factor and its standard deviation as obtained from the general similarity transformation model using the program as described in (Kumar, 1972) were

$$\lambda = 1 + 5.92 \times 10^{-6}$$

$$\sigma_\lambda = 3.54 \times 10^{-6}$$

A separate adjustment program has been written to obtain the scale factor from chords (4-22). The result (in perfect agreement with the above one) was

$$\lambda = 1 + 5.9181 \times 10^{-6}$$

$$\sigma_\lambda = 3.5393 \times 10^{-6}$$

Different independent set of chords yielded the same values.

However, the weighted mean method using all possible chords as described in (Kumar, 1972) yielded:

$$\lambda = 1 + 6.20 \times 10^{-6}$$

$$\sigma_\lambda = 1.71 \times 10^{-6}$$

The standard deviation is much too optimistic.

5. DISTORTIONS IN GEODETIC DATUMS FROM SIMILARITY TRANSFORMATIONS

5.1 Procedure

During recent years geocentric coordinates have been determined in many parts of the world from satellite Doppler tracking. These coordinates are known to be very accurate and homogeneous and are based on the precise ephemeris furnished by the Naval Surface Weapon Center. These coordinates constitute a good comparison standard with respect to the geodetic coordinates.

The procedure used in this analysis is as follows. A transformation between the Cartesian coordinates (X) from the Doppler solution and the geodetic coordinates (U) is carried out. The geodetic Cartesian coordinates are obtained from conversion of the geodetic latitude ϕ, the geodetic longitude λ, the orthometric height H, and the geoid height N_G. If h is the height above the ellipsoid, e.g.,

$$h = H + N_G \tag{5-1}$$

then

$$u = (N + h) \cos \phi \cos \lambda \tag{5-2}$$

$$v = (N + h) \cos \phi \sin \lambda \tag{5-3}$$

$$w = (\frac{b^2}{a^2} N + h) \sin \phi \tag{5-4}$$

where a and b are the semimajor and semiminor axes of the reference ellipsoid, and N is the radius of curvature in the prime vertical. After determining the transformation parameters between the two systems from a least squares solution, the systematic differences between the two sets of coordinates should be absorbed by the transformation parameters, and the residuals should appear randomly provided that

(a) the mathematical model for the transformation is correct,

(b) there are no patterned distortions in either set of coordinates.

Thus, the only task remaining is that of scanning the residuals for systematic behavior. This is most efficiently done by representing the differences between the residuals for each coordinate of the same station in a map. It is then possible to interpolate so-called iso-residual-difference-lines which will demonstrate the systematic behavior of the residual differences adequately. In principle, three maps can be plotted for each datum and transformation model. In order to simplify the interpretation, the residual differences are expressed as residual differences in geodetic coordinates ϕ, λ, h instead of Cartesian coordinates. The relation between these types of residuals is given by

$$\begin{bmatrix} (M+h)\ V_\phi \\ (N+h)\cos\phi\ V_\lambda \\ V_h \end{bmatrix} = \begin{bmatrix} -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \qquad (5\text{-}5)$$

5.1.1 Weighting of Observations

The Cartesian coordinates of the satellite and the geodetic systems are taken as observations. The procedure described above is valid rigorously only when the full variance-covariance matrices for the two systems are available. Generally this is not the case. The situation for this study is as follows:

(1) The correlations between the coordinates of various stations in the geodetic system are not known. Also, the covariance matrix for the set of satellite coordinates is not available.

(2) The correlation between the coordinates of the same station cannot be established reliably. For the Doppler coordinates only the statistics

reflecting the internal accuracy of the position are available. These statistics cannot be used as an assessment of accuracy. It was, therefore, decided to use in all computations an estimated accuracy of ± 1.5 m for each of the coordinates. The accuracy of the geodetic coordinates was estimated from the empirical formulas of Simmons (1950).

At a first glance the value of the whole investigation may seem to be in doubt because of the lack of appropriate statistics. It is true that it is difficult to assess the effect of not having the correlation between stations; however, once a diagonal weight matrix had been accepted, it was verified numerically and theoretically that the resulting residual differences are relatively insensitive with respect to the selection of the variances for the station coordinates. Using the weight matrix as expressed in (3-64), and using the notation of Section 3, it is easily verified that equation (3-55) becomes

$$X = -\sum_{j=1}^{r}\left[\sum_{i=1}^{r}\{A_i^T(\bar{k}\,\Sigma_{U_i} + \Sigma_{X_i})^{-1}A_i\}\right]^{-1} A_j^T(\bar{k}\,\Sigma_{U_i} + \Sigma_{X_i})^{-1}W_j \tag{5-6}$$

$\bar{k}$ is a factor which allows the scaling of the variances of the geodetic coordinates with respect to those of the satellite coordinates. Extensive trial computations were performed with reasonable variation of $\bar{k}$ and different selections of Σ_U. It was found that the parameters X do not change significantly (less than once the standard deviation). $\bar{k}$ was finally selected such that the variance of unit weight $\hat{\sigma}_0^2$ was close to unity for a seven-parameter transformation.

For this investigation it is also important to see how the residuals are effected by the variations in weight. The corresponding equations are

$$V_{U_i} = -\bar{k}\, \Sigma_{U_i} (\bar{k} \Sigma_{U_i} + \Sigma_{X_i})^{-1} \; (A_i X + W_i) \tag{5-7}$$

$$V_{X_i} = \Sigma_{X_i} (\bar{k} \Sigma_{U_i} + \Sigma_{X_i})^{-1} \; (A_i X + W_i) \tag{5-8}$$

and

$$V_{X_i} - V_{U_i} = A_i X + W_i \tag{5-9}$$

Equation (5-9) shows that the residual differences depend only indirectly through the parameters X on the selection of the variances. It was found empirically that the residual differences alter by only a few decimeters when changing the variances within reasonable limits. This amount is well within the required accuracy of this analysis.

5.2 Distortions in the North American Datum (NAD 1927)

There are a great number of Doppler stations available on the North American continent. The vast majority of station coordinates are given in the system NWL 10E - 9D which is to be interpreted as the 9D-datum and the 10E-gravity model. The remaining few stations are from solution NWL 9B-9D. The 10E gravity model which is used operationally is actually an augmented version of the model 10 D07 which, in turn, is the successor of gravity model 9B (Anderle, 1975). Both sets of station coordinates can be combined in the analysis without losing accuracy. It is, however, important to realize that the coordinates given are characteristically different from the set known as NWL-10F which aims to establish the correspondence of the Doppler network with terrestrial and VLBI surveys. The heights were altered to accommodate the scale discrepancy. It also includes a small longitudinal rotation to match the latest gravity observations

in the North American Datum (since the Doppler longitude is arbitrary).

The coordinates in the NWL 10F system are computed from the NWL 9D coordinates as follows:

$$\phi' = \phi$$

$$\lambda' = \lambda + 0\overset{''}{.}26 \quad (\lambda \text{ east is positive})$$

$$h' = h - 5.27 \text{ m}$$

where h and h' are heights above a selected common ellipsoid.

The NAD 1927 positions form the second set of coordinates to be used in this comparison study. The heights above the reference ellipsoid are obtained from the orthometric heights for the station mark plus the geoid heights which refer to the astrogeodetic geoid having zero undulation at the initial point of triangulation, e.g., Meades Ranch.

As explained earlier, a variance of 2.25 m^2 was assumed for each of the Doppler coordinates. The variances of the NAD 27 coordinates were computed according to the empirical formula by Simmons (1950).

$$\sigma_U^2(m^2) = \left(\frac{1}{20\,000 \sqrt[3]{M_{(miles)}}} \cdot M_{(m)} \right)^2 \tag{5-10}$$

where M is the distance from the initial point of the triangulation (Meades Ranch). These variances were scaled by a factor $\bar{k}$ such that the variance of unit weight $\hat{\sigma}_o^2$ was close to unity. $\bar{k}$ was found to be

$$\bar{k} \approx 0.16$$

This value might indicate that Simmons' rule yields somewhat too pessimistic variances. But it should be remembered that Simmons' rule actually is meant to give the accuracy of distances between points and not the accuracy for each of the coordinates.

A summary of transformation parameters is given in Table 5-1. Other tables showing the correlation coefficients are Tables 5-3 through 5-7.

Figs. 5-1 through 5-3 are the corresponding residual maps.

It has been argued for quite a long time that there are significant scale variations within the NAD 1927. To estimate these variations an empirical method was devised which associates a scale distortion number with each station P_i. This number is obtained by computing an average station scale factor from chord comparisons, using chords starting at P_i. The method is strongly dependent on the choice of the lengths of the chords used in the comparison. The use of long chords weakens the representation of "local" distortions. A map would show a smooth picture. In contrast, the use of only short chords creates an unrealistically rough and inaccurate representation. It was, therefore, decided to attempt the following two versions:

(1) All the chords starting at P_i up to a length of 1500 km were compared and averaged (Fig. 5-4).

(2) All the chords starting at P_i and having a length between 1000 km and 2000 km were compared and averaged (Fig. 5-5).

Both maps indicate a significant scale difference for the Eastern and Western halves of the United States. When trying to assess the accuracy of these two maps one should remember the limitations in accuracy which are inherent in the method of their design. They might be useful for scale modeling. In this investigation they were only used to justify the splitting up of the total area into an Eastern part and a Western part, roughly along the longitude of Meades Ranch.

Subsequently, the same procedure as described for the NAD 1927 was used separately for the Eastern and the Western parts. The results are given in Table 5-1, Tables 5-8 through 5-11, and Figs. 5-6 through 5-11. The

results convincingly confirm the existence of large scale variations between the two parts of the NAD 1927. Also, quite significant differences in orientation could be detected.

5.3 Distortions in the North American Datum (NAD - MR 1972)

Although rather unlikely, the distortions found in the previous comparisons might be attributed equally to both the Doppler system and the NAD 1927 coordinates. To clarify which contributes more, another comparison was made between the Doppler stations and the stations on the NGS transcontinental precise traverse, which are known to be of superior quality. The geodetic positions of the stations, referenced to the NAD - MR 1972 Datum, were obtained from adjustments as follows (Meade, 1975):

(1) Western loop adjustment of the transcontinental traverse. This loop involves stations 10006-10018-51103-30098-30099-10006. The NAD 1927 position of Meades Ranch was used for position control.

(2) Eastern loop adjustment of the transcontinental traverse. This loop involves stations 10006-10019-20001-51068-20016-10018-10006. Also the loop involving station 10003 and section from 20016 to junction near 10019. The NAD 1927 position of Meades Ranch was used as position control.

(3) Northeastern section of the western loop from junction north of Meades Ranch to 51044 to 30099. The junction point north of Meades Ranch and station 30099, as determined in the western loop adjustment, were used as position control.

(4) Positions of stations 51014-51015; 51048; 10031; and 10055 were determined from spur adjustment with control from the main traverse loops.

(5) Stations 10018, 51067 and 51030 are common to the eastern and western loops. The positions given for these stations are the mean values of results from the eastern and western loop adjustments.

The NAD - MR 1972 positions are considered as preliminary. A simultaneous adjustment of the traverse net will be performed after the field surveys are completed.

The results of the transformations using equal weights are given in Table 5-1, Tables 5-16 through 5-19, and Figs. 5-12 and 5-13. The scale distortion maps are given in Figs. 5-14 and 5-15. There are no indications of any distortions between the NAD - MR 1972 system and the Doppler system.

5.4 Distortions in the Australian Geodetic Datum (Aus)

The same procedure as described earlier has been followed. The satellite data are from solution NWL 8F-9D. The heights in the geodetic data are obtained from the Australian Height Datum and the 1971 geoid determination (Bomford, 1974). The results are given in Tables 5-2, Tables 5-20 through 5-23, and Figs 5-16 through 5-17.

5.5 Distortions in the South American Datum (SAD 69)

The investigations on the distortion in the SAD 69 system are strongly affected by the limited number of Doppler stations available and by their distribution. The station positions are referenced to the NWL 9D station set and 10E gravity model. The datum used for the geoid heights is the SAD 69 with N = 0 at CHUA. The results are given in Table 5-2, Tables 5-24 through 5-27, and Figs 5-18 and 5-19.

5.6 Tables and Figures

Guide Matrix for Locating Figures and Tables of Section 5

Type of Map (Table)	No. of Parameters	Model	Systems: NAD 1927	NAD 1927-E	NAD 1927-W	Precise Traverse	AUS	SAD 69
Distortion Maps (Residuals)	3	--	5-1	6	9	12	16	18
	4	--	2	7	10	--	--	--
(Fig. No.)	7	--	3	8	11	13	17	19
Distortion Maps (Scale) (Fig. No.)	--	--	4, 5			14, 15		
Transformation Parameters and Residuals	3	--	5-3	8	12	16	20	24
	4	Bursa	4	9	13	--	--	--
(Table No.)	7	Bursa	5	10	14	17	21	25
	7	M-Badekas	6	11	15	18	22	26
	7	Veis	7	--	--	19	23	27
Summary of Transformation Parameters (Table No.)	--	--	1	1	1	1	2	2

The geodetic and satellite determined coordinates used in these transformations may be found in the 15th and 17th Semi-Annual Reports on Grant No. NGL 36-008-093, prepared for NASA Headquarters, Washington, D.C.

Table 5-1

Summary of Transformation Parameters: System to NWL 9D

System:	NAD - MR 1972			NAD 1927			
No. of Points:	44	44	44	76			
Transformation Model:	B_3, M_3	B_7	M_7[1]	B_3, M_3	B_4	B_7	M_7[1]
Δx (m)	-26.8±0.3	-23.6±1.3	-26.7±0.2	-28.1±0.4	-26.9±0.4	-21.1±1.9	-28.4±0.3
Δy (m)	151.4±0.3	155.7±1.6	151.5±0.2	152.8±0.4	163.3±1.4	166.2±2.6	152.9±0.4
Δz (m)	177.8±0.3	173.4±2.1	178.1±0.2	178.5±0.4	170.4±1.1	175.1±3.1	179.1±0.3
$\Delta \times 10^6$		1.03±0.16	same as previous column		2.13±0.28	2.13±0.27	same as previous column
ω (")[2]		0.15±0.04				0.29±0.06	
ψ (")[2]		-0.05±0.04				-0.06±0.06	
ε (")[2]		0.02±0.08				-0.20±0.12	
α (")[3]		0.13±0.03				0.26±0.06	
ξ (")[3]		0.02±0.08				-0.19±0.12	
ε (")[3]		-0.08±0.04				-0.17±0.06	

[1] These parameters are given for comparison with earlier solutions.

[2] ω, ψ, ε when positive, represent counterclockwise rotations about the w,v,u axes, as viewed from the end of the positive axis.

[3] The rotations α, ζ, ε refer to the Veis model and are explained in section 2.3

Table 5-1 (cont'd)

Summary of Transformation Parameters: System to NWL 9D

System:	NAD 1927 EAST				NAD 1927 WEST			
No. of Points:	45				31			
Transformation Model:	B_3, M_3	B_4	B_7	M_7[1]	B_3, M_3	B_4	B_7	M_7[1]
Δx (m)	-27.9±0.4	-28.1±0.4	-25.9±3.4	-27.4±0.4	-28.5±0.7	-18.7±0.9	-12.9±2.9	-28.6±0.3
Δy (m)	151.1±0.4	160.5±2.4	161.4±3.3	151.6±0.4	155.2±0.7	183.1±2.4	180.2±2.6	155.5±0.3
Δz (m)	177.5±0.4	170.7±1.7	171.9±3.8	178.5±0.4	180.0±0.7	156.7±2.0	155.7±2.3	180.6±0.3
$\Delta \times 10^6$		1.84±0.45	1.84±0.40	same as previous column		5.95±0.5	5.95±0.34	same as previous column
ω (")[2]			-0.24±0.10				0.60±0.9	
ψ (")[2]			0.46±0.10				-0.41±0.08	
ε (")[2]			-0.06±0.14				-0.10±0.10	
α (")[3]								
ξ (")[3]								
ε (")[3]								

[1]These parameters are given for comparison with earlier solutions.
[2]ω, ψ, ε when positive represent counterclockwise rotations about the w, v, u axes, as viewed from the end of the positive axis.
[3]The rotations α, ξ, ε refer to the Veis model and are explained in section 2.3.

Table 5-2

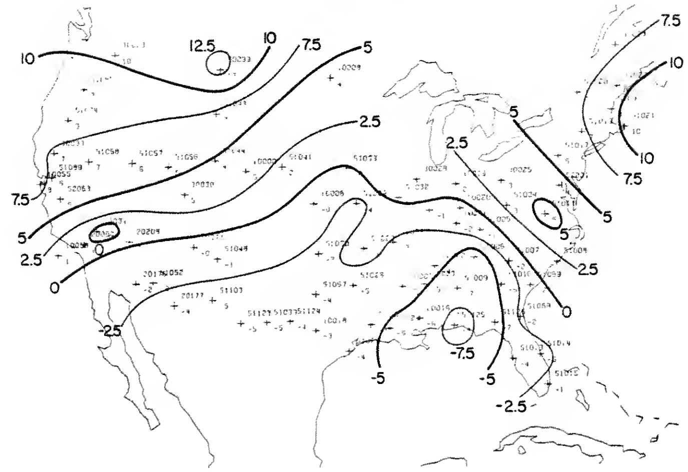
Summary of Transformation Parameters: System to NWL 9D

System:	SAD 69			AUS		
No. of Points:	9			7		
Transformation Model:	B_3, M_3	B_7	M_7	B_3, M_3	B_7	M_7
Δx (m)	-80.4±2.6	-39.4±24.6	-77.8±4.7	-123.1±1.5	-115.5±7.2	-122.4±1.4
Δy (m)	- 0.3±2.6	7.8±11.8	-12.4±8.6	- 30.9±1.5	- 28.7±7.0	- 30.4±1.4
Δz (m)	-40.3±2.6	-37.4± 7.6	-49.5±4.2	141.2±1.5	138.1±8.3	140.0±1.4
$\Delta \times 10^6$		-0.99±1.16	same as previous column		0.36±0.91	same as previous column
ω (")		1.18±0.86			0.07±0.24	
ψ (")		-0.90±0.33			-0.52±0.25	
ε (")		0.16±0.26			0.35±0.25	
α (")		0.33±0.25			-0.59±0.19	
ξ (")		-0.48±0.26			0.10±0.29	
ε (")		-1.37±0.89			0.20±0.25	

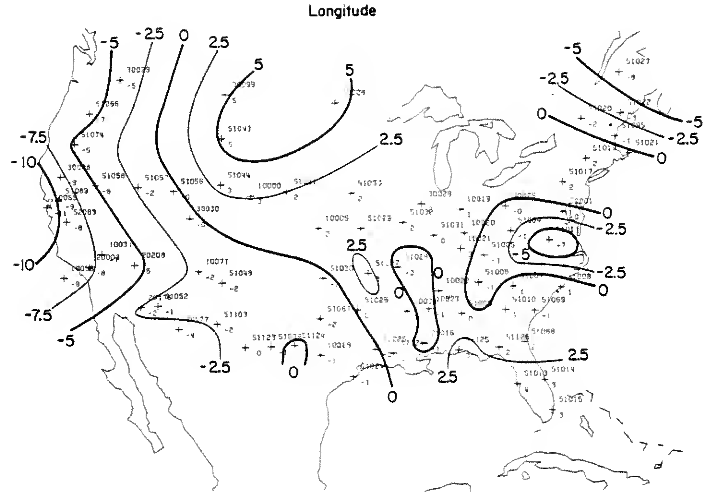
Fig. 5-1

Differences After Transforming NAD27 to NWL9D
(in Metres) 3 Transformation Parameters
(NWL9D to NAD27)

Latitude



Longitude



Height

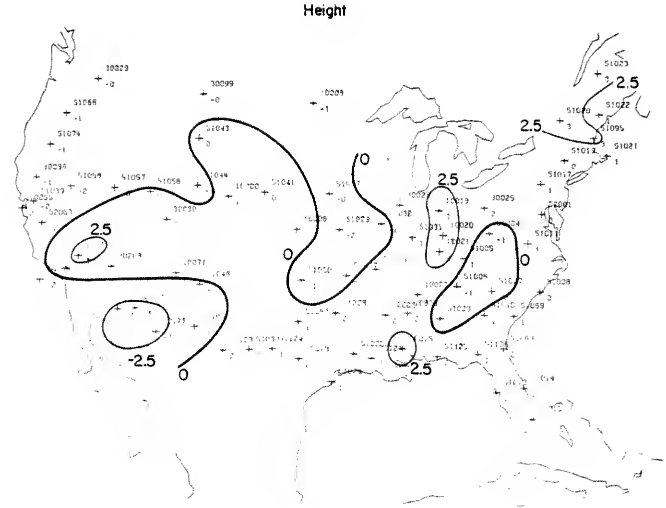
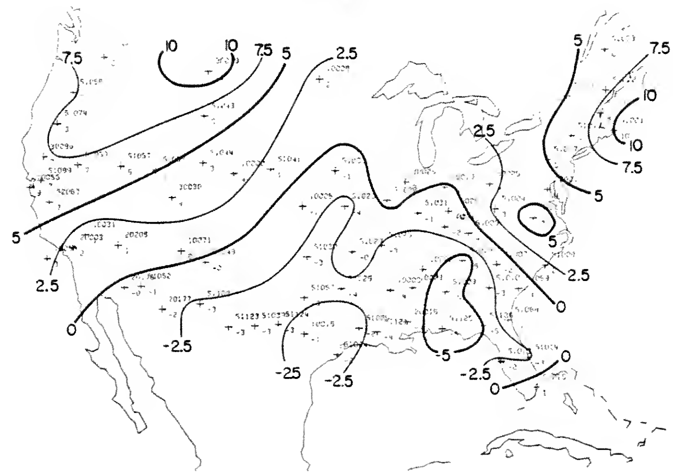


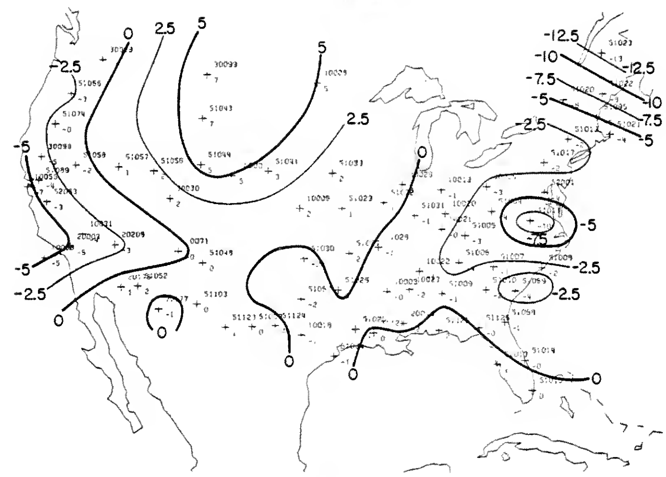
Fig. 5-2

Differences After Transforming NAD27 to NWL9D
(in Metres) 4 Transformation Parameters
(NWL9D to NAD27)

Latitude



Longitude



Height

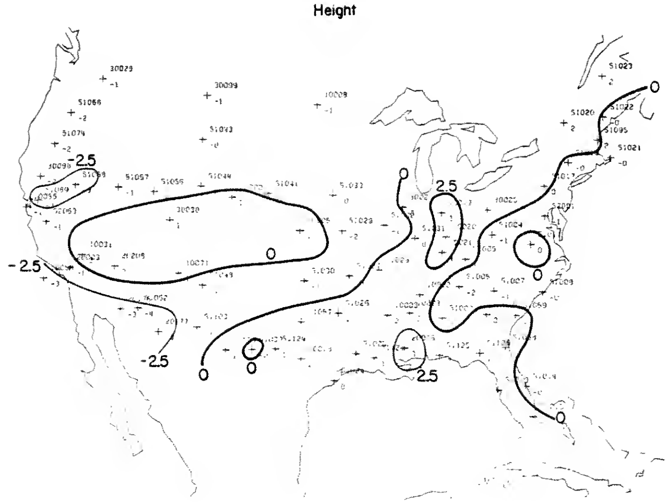
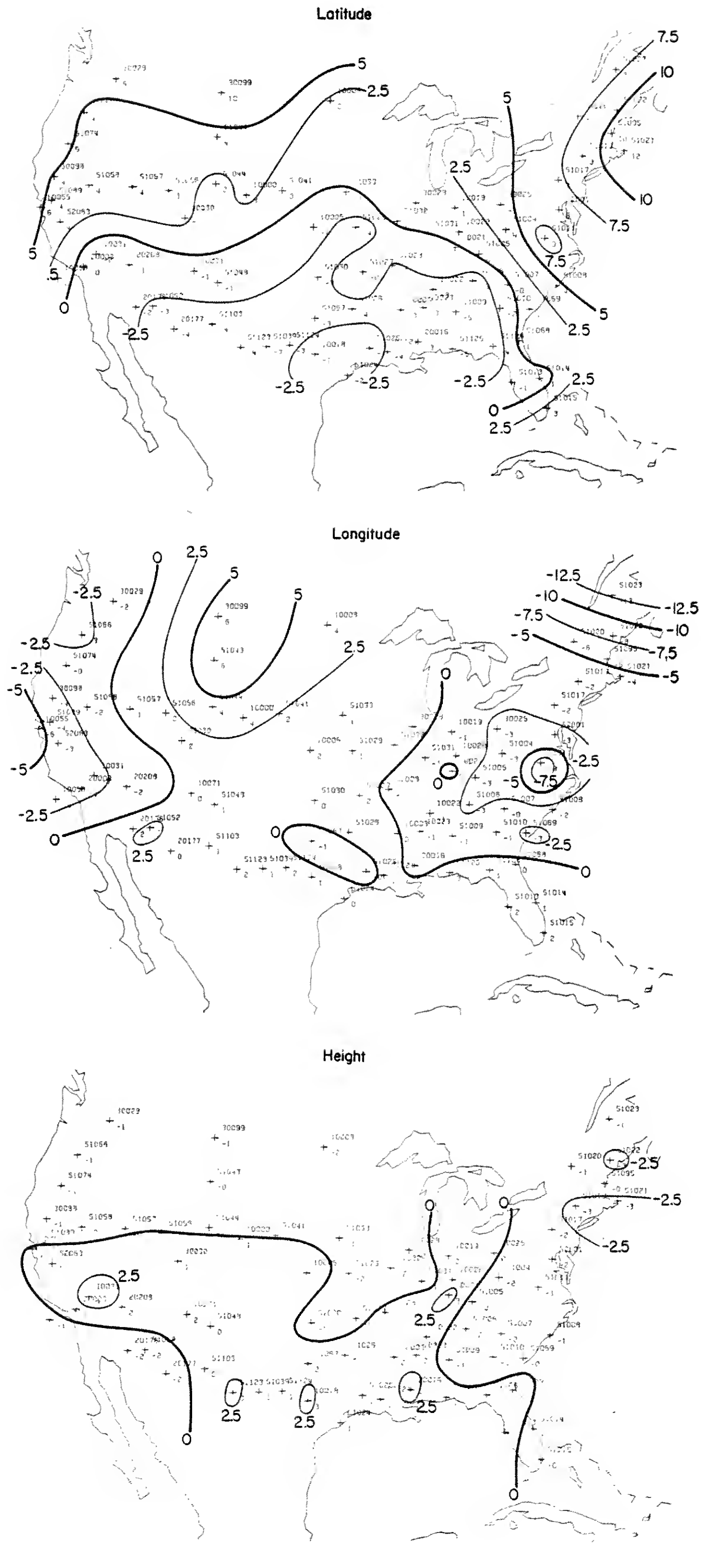


Fig. 5-3

Differences After Transforming NAD27 to NWL9D
(in Metres) 7 Transformation Parameters
(NWL9D to NAD27)



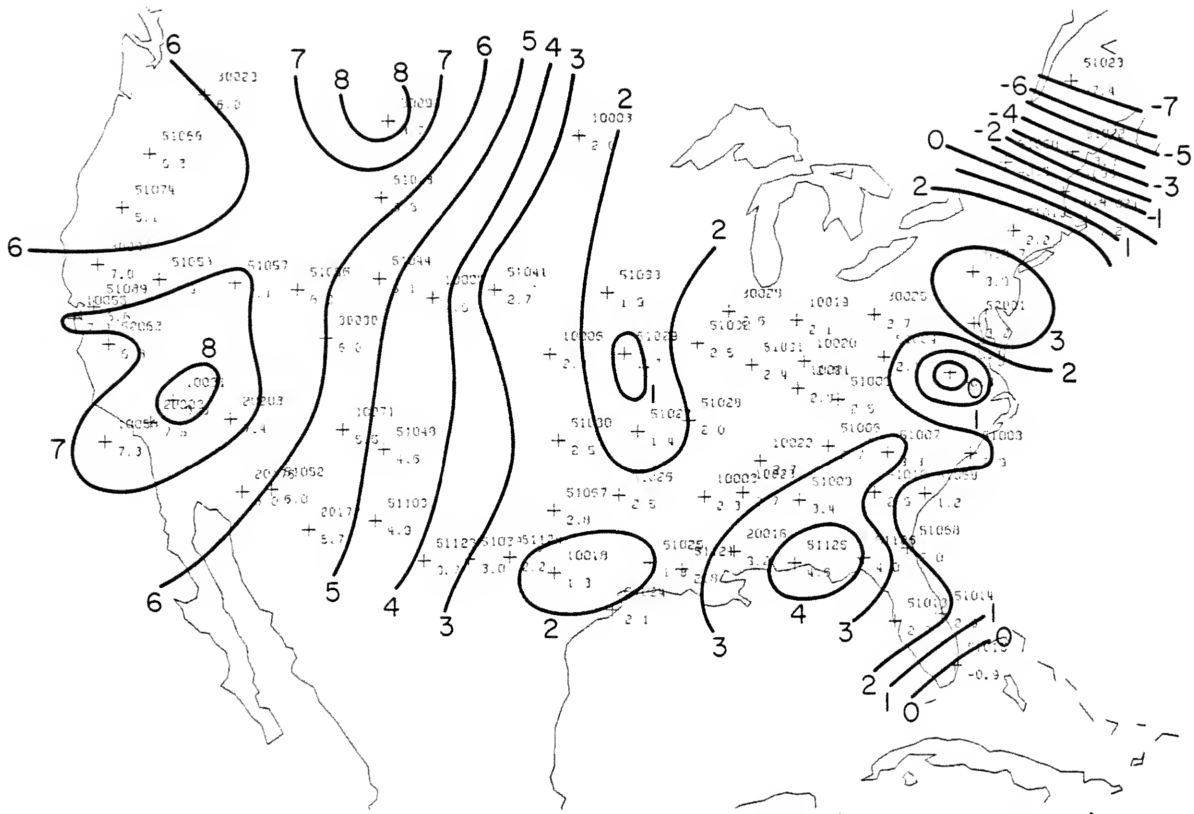


Fig. 5-4 Scale distortions on the NAD 1927 (chord < 1500 km)

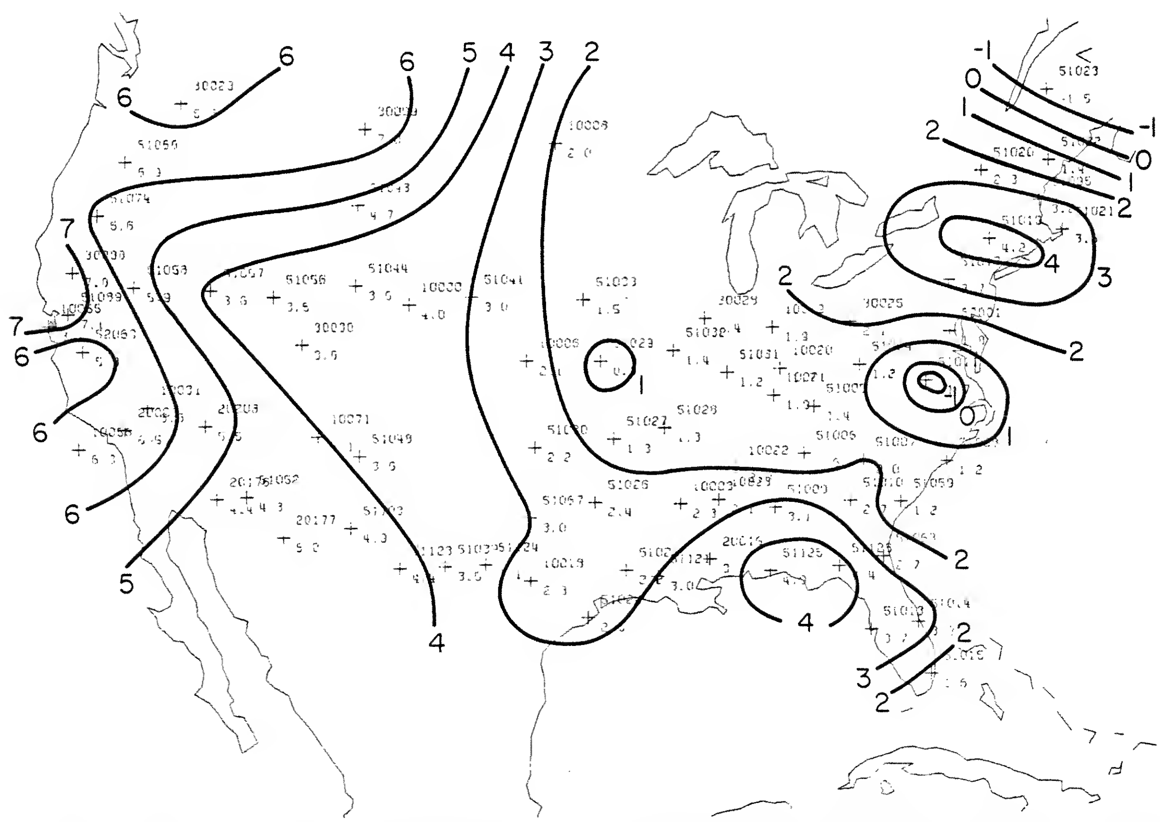


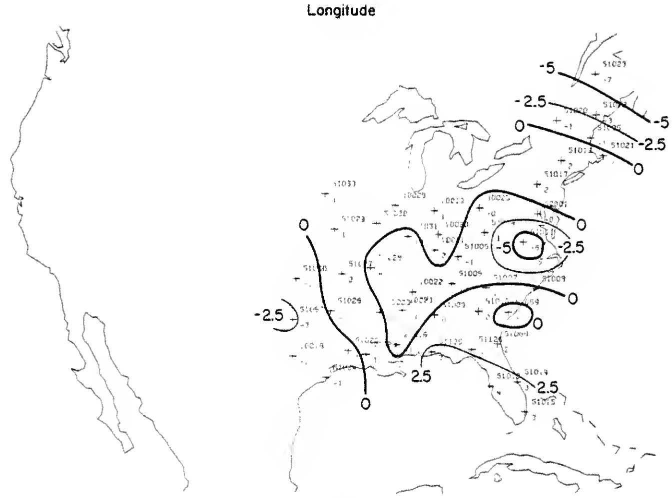
Fig. 5-5 Scale distortions on the NAD 1927 (1000 km < chord < 2000 km)

Fig. 5-6

Differences After Transforming NAD27-East to NWL9D
(in Metres) 3 Transformation Parameters
(NWL9D to NAD27)

Latitude

Longitude



Height

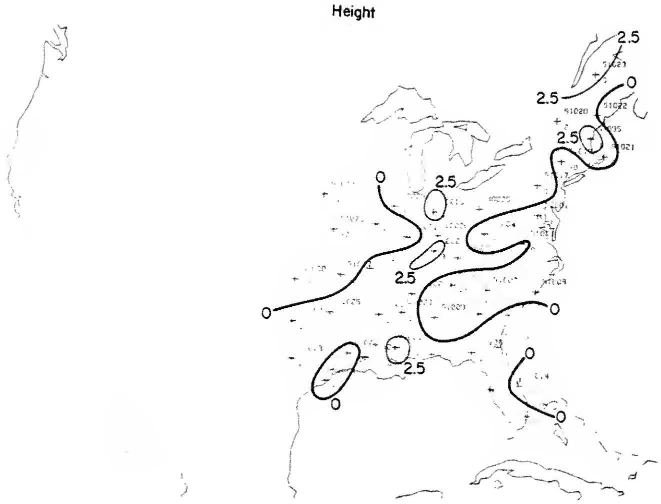
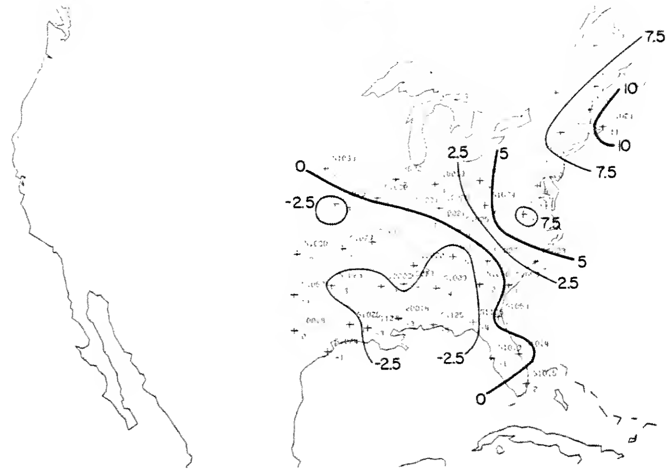


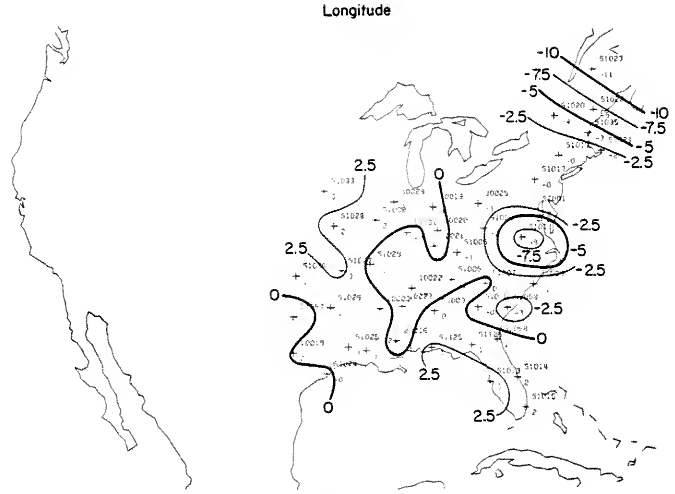
Fig. 5-7

Differences After Transforming NAD27-East to NWL9D
(in Metres) 4 Transformation Parameters
(NWL9D to NAD27)

Latitude



Longitude



Height

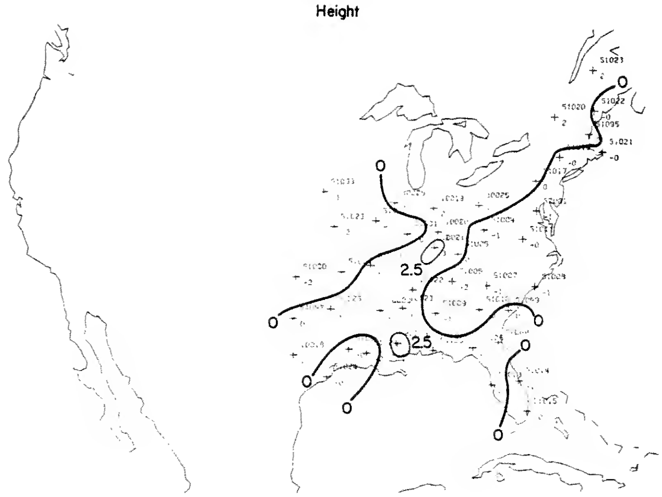


Fig. 5-8

Differences After Transforming NAD27-East to NWL9D
(in Metres) 7 Transformation Parameters
(NWL9D ~ NAD27)

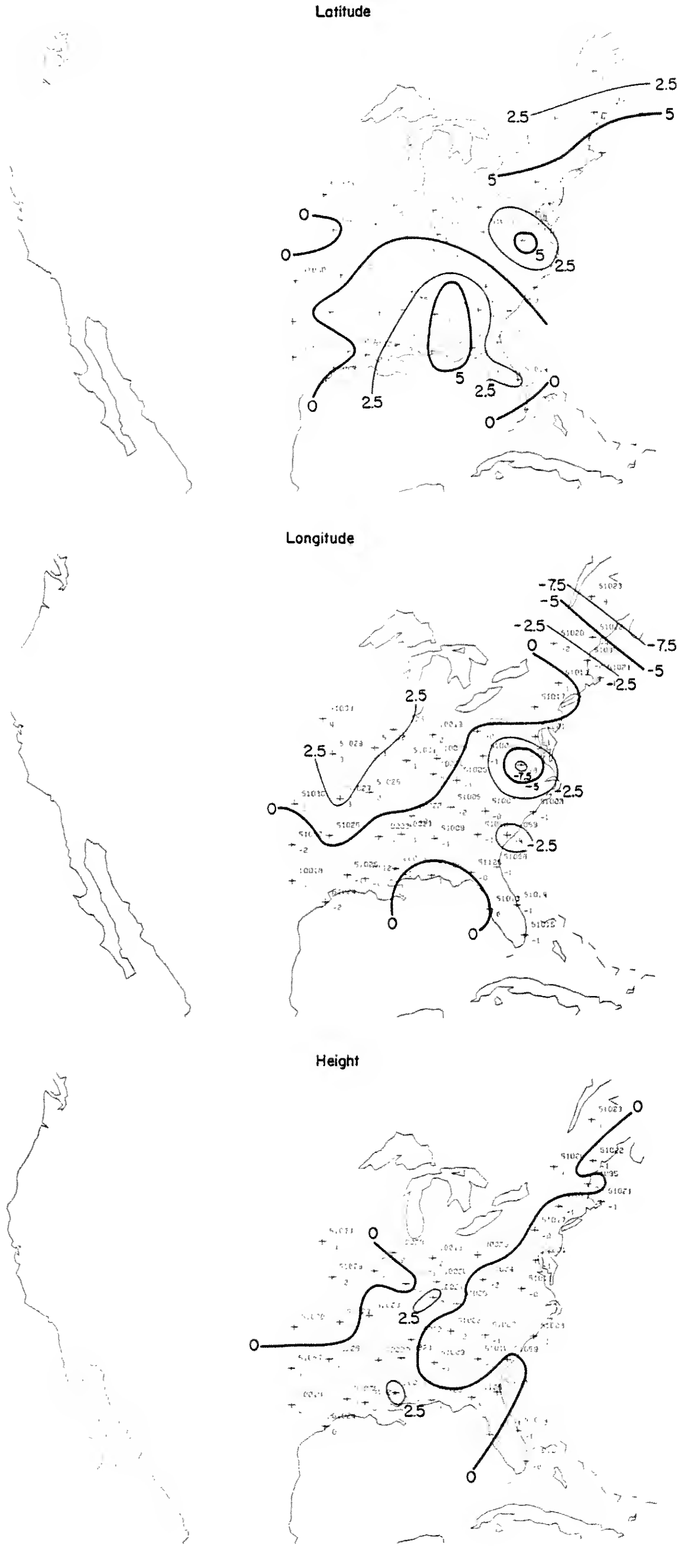
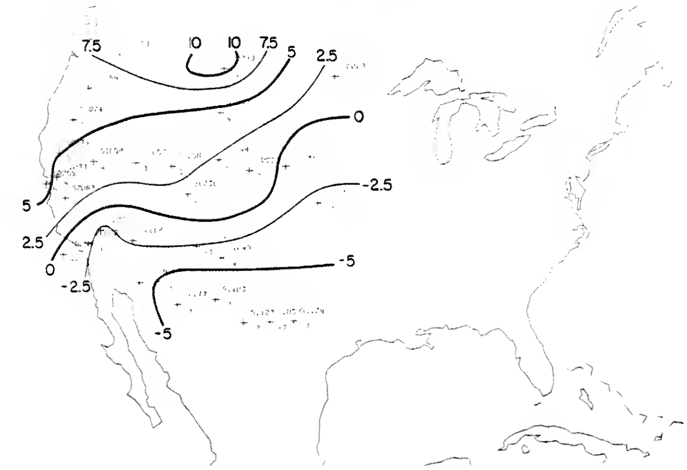


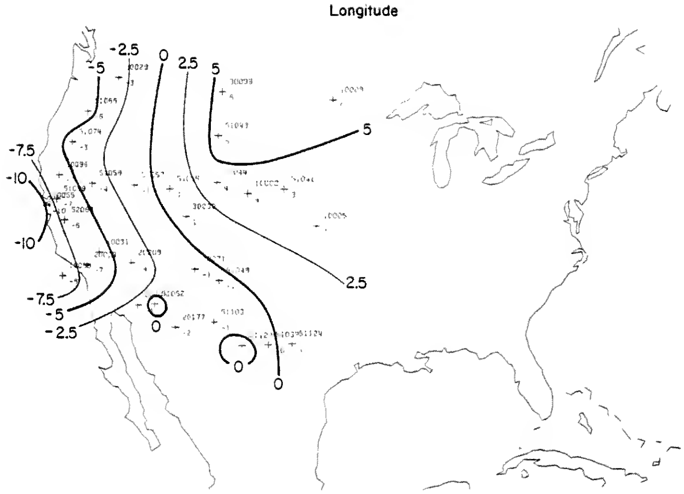
Fig. 5-9

Differences After Transforming NAD27-West to NWL9D
(in Metres) 3 Transformation Parameters
(NWL9D to NAD27)

Latitude



Longitude



Height

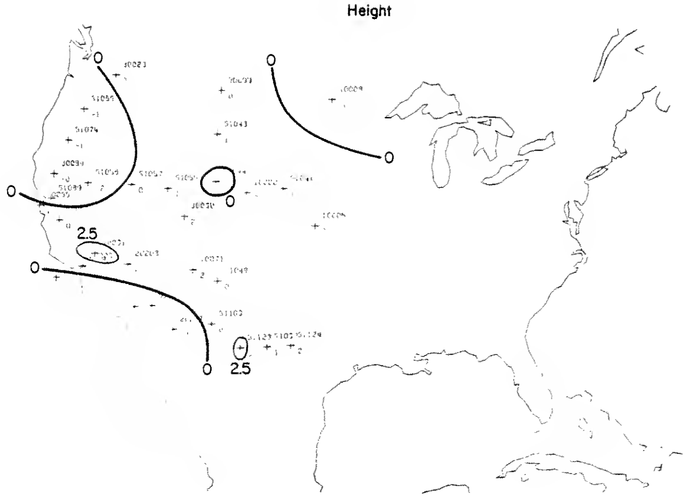
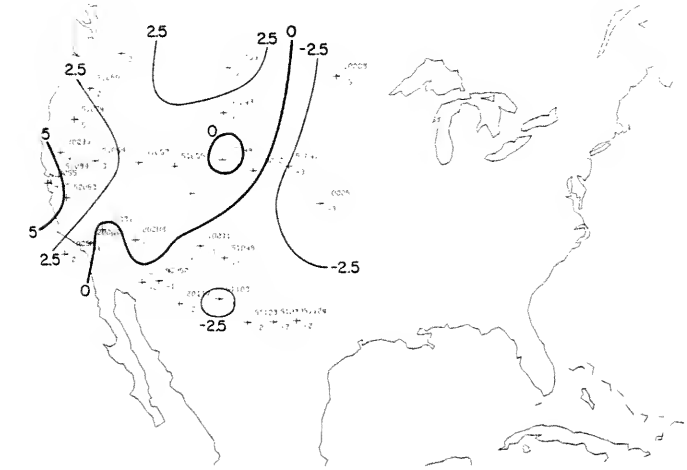


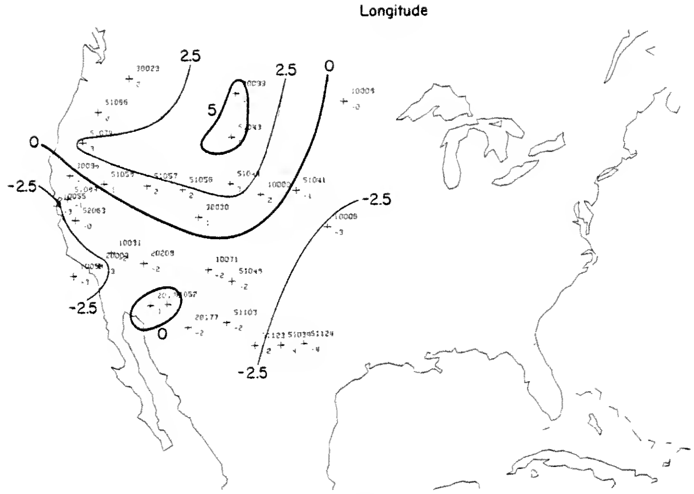
Figure 5-10

Differences After Transforming NAD27-West to NWL9D (in Metres) 4 Transformation Parameters (NWL9D to NAD27)

Latitude



Longitude



Height

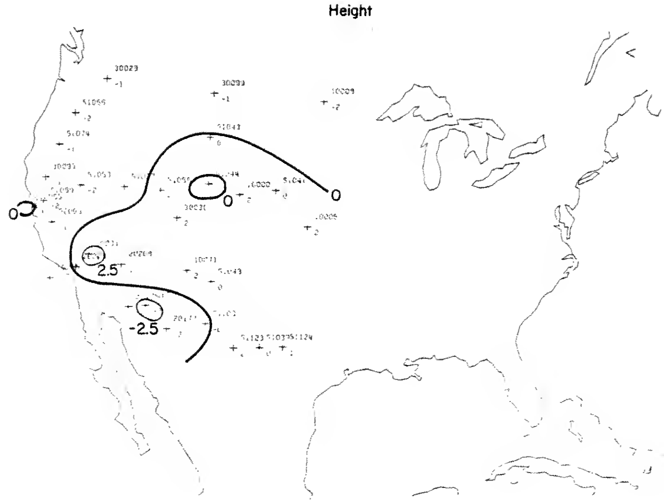


Fig. 5-11

Differences After Transforming NAD27-West to NWL9D
(in Metres) 7 Transformation Parameters
(NWL9D — NAD27)

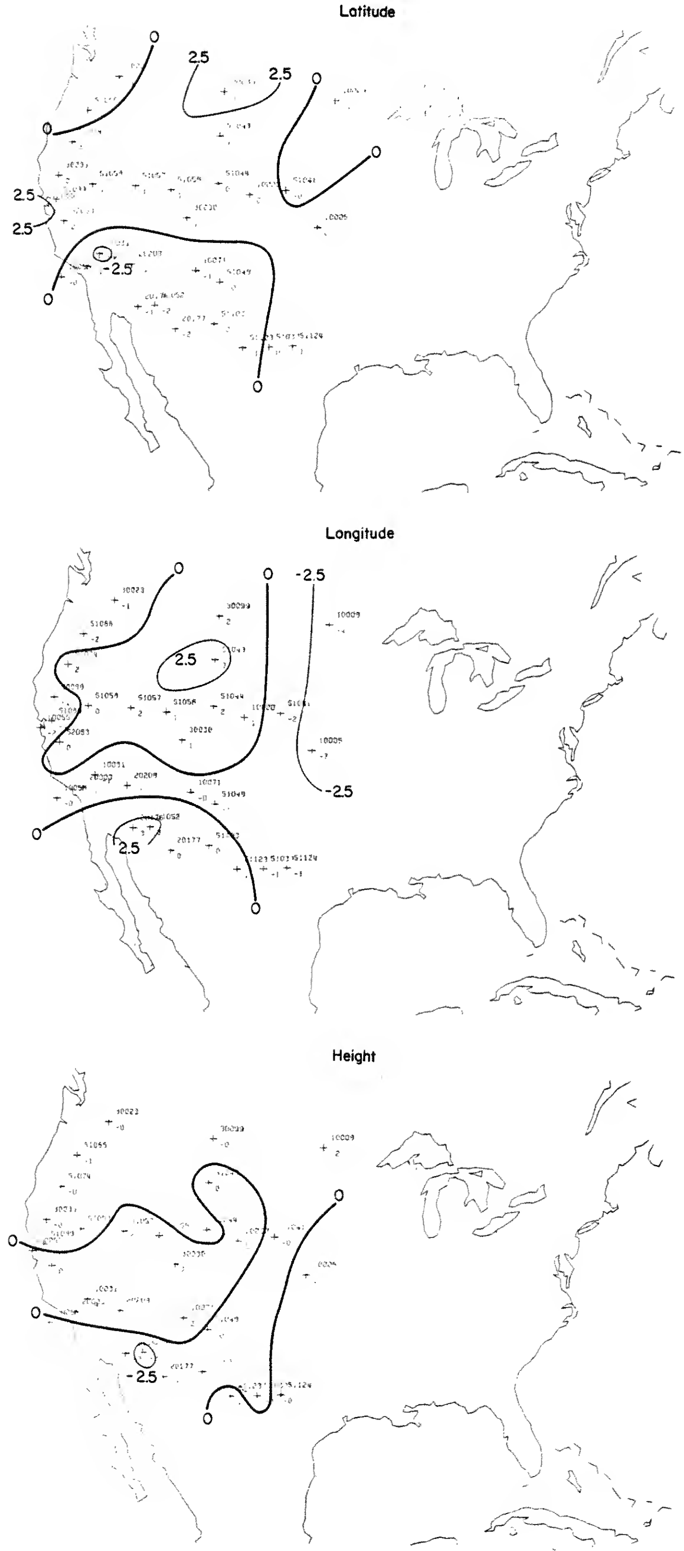
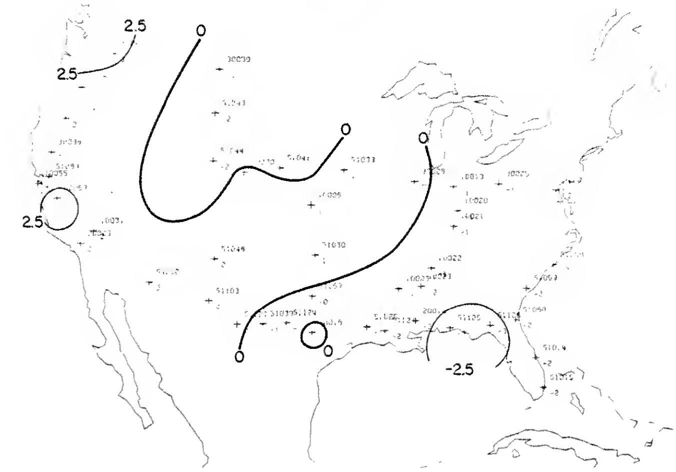


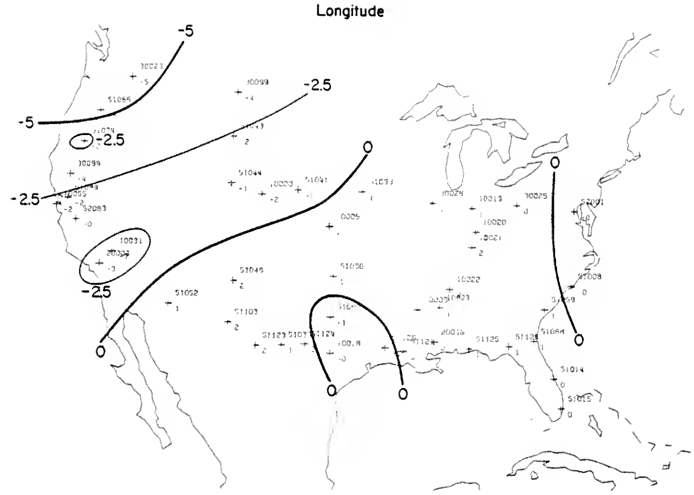
Fig. 5-12

Differences After Transforming Precise Traverse to NWL9D
(in Metres) 3 Transformation Parameters
(NWL9D - Precise Traverse)

Latitude



Longitude



Height

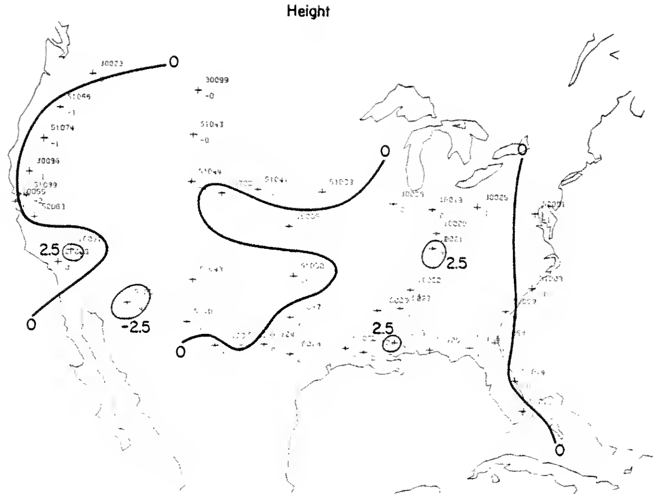
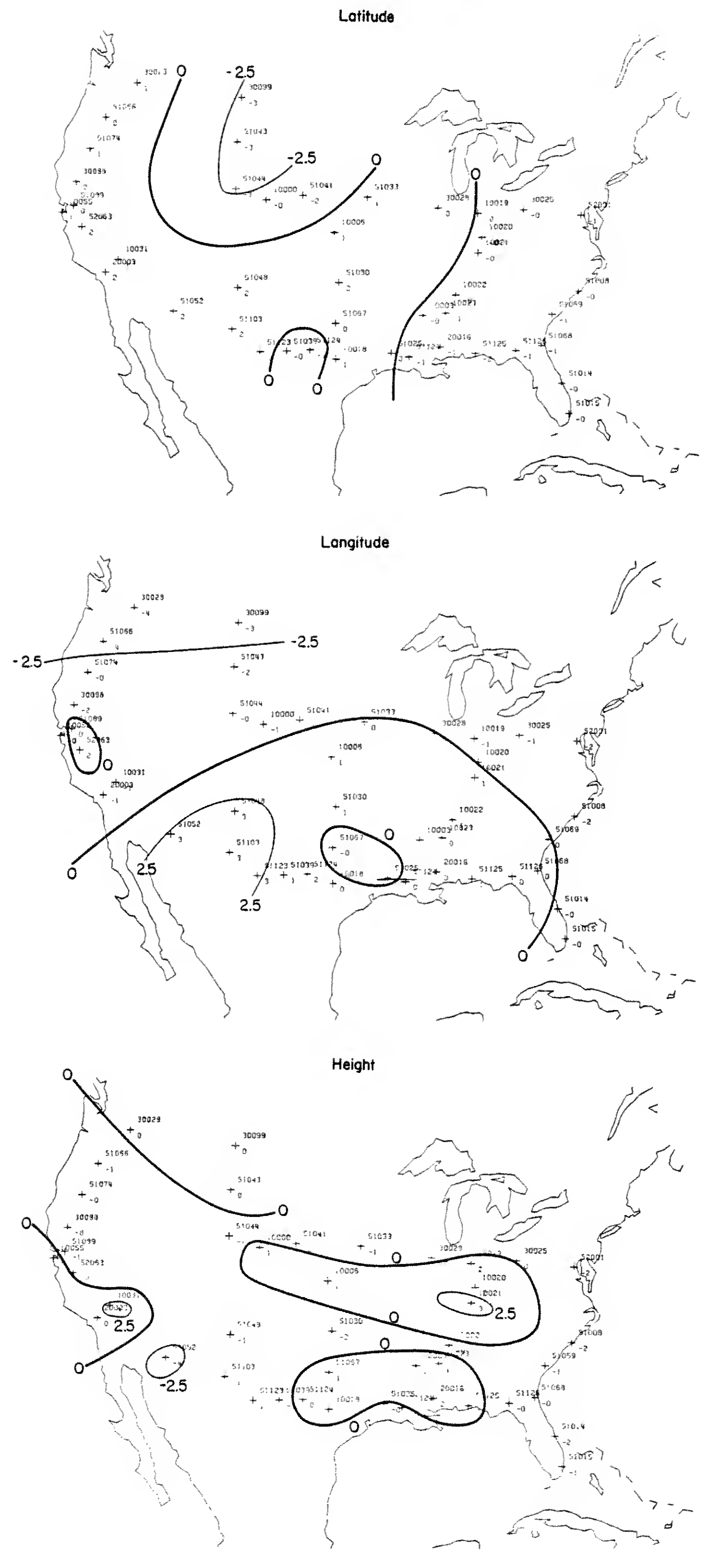


Fig. 5-13

Differences After Transforming Precise Traverse to NWL9D
(in Metres) 7 Transformation Parameters
(NWL9D - Precise Traverse)



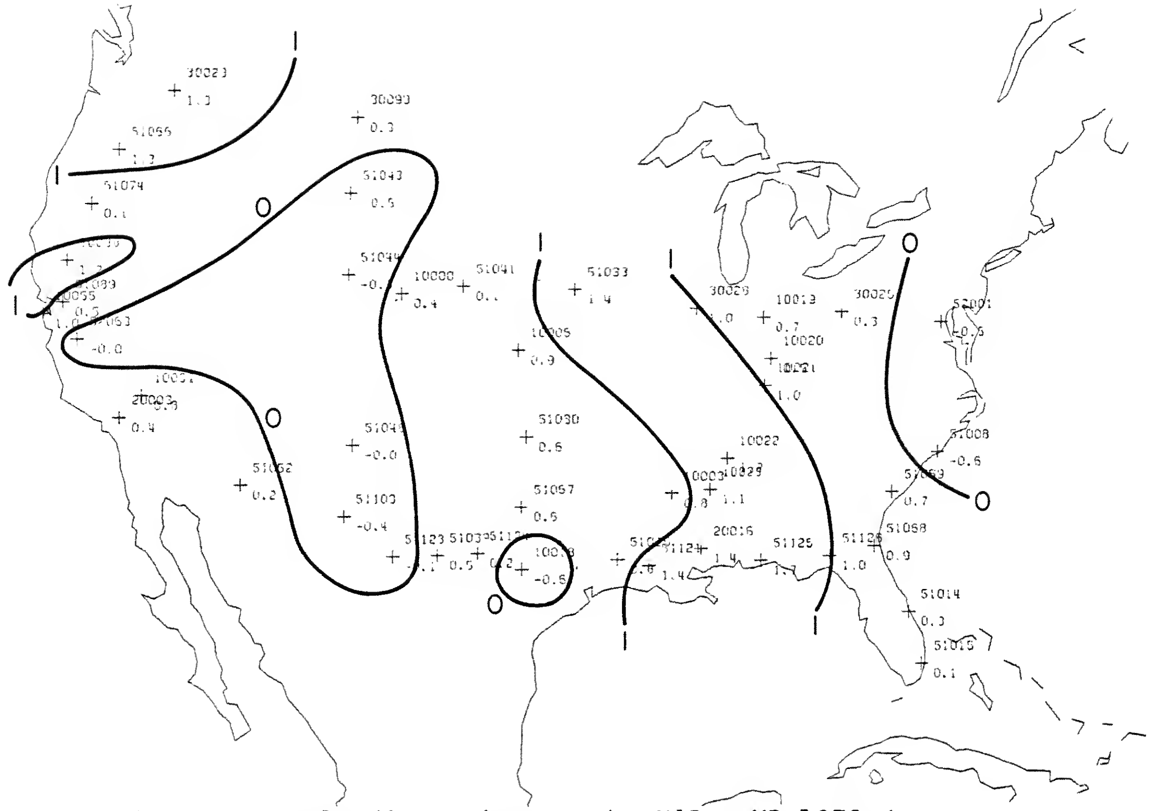


Fig. 5-14 Scale distortion on the NAD - MR 1972 datum (chord < 1500 km)

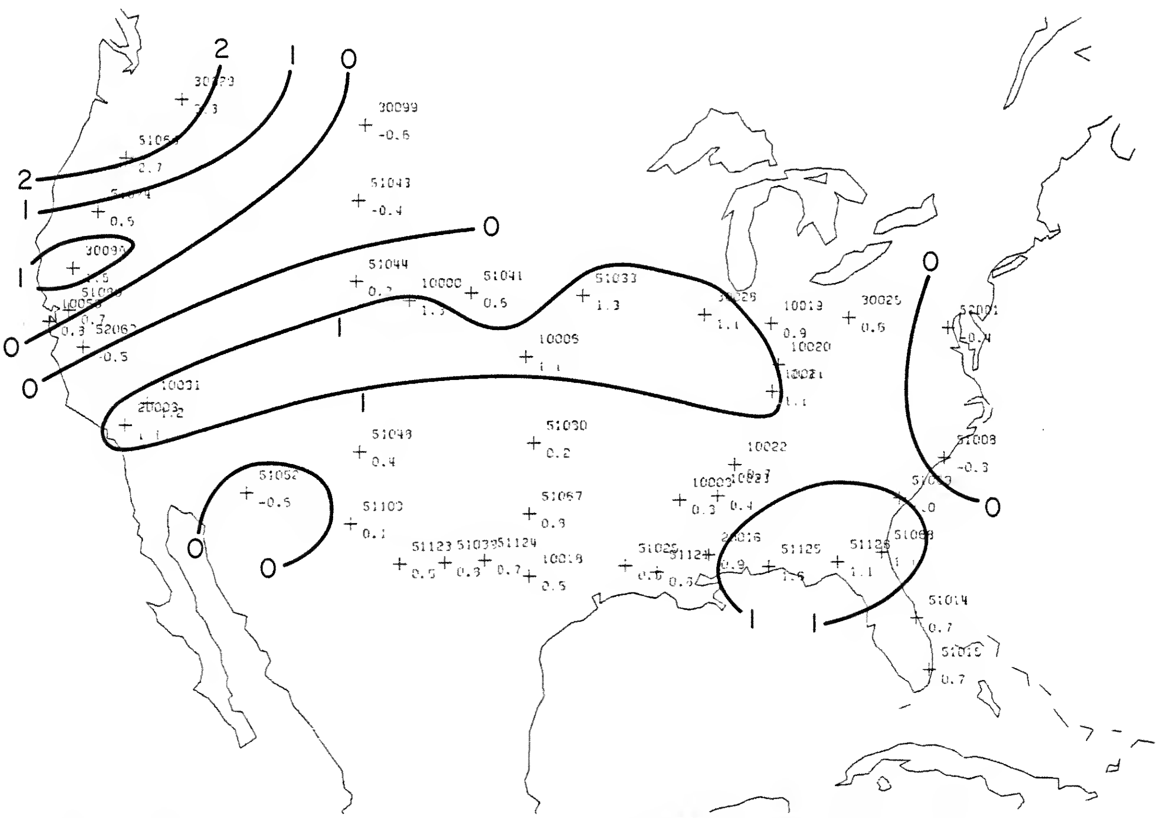


Fig. 5-15 Scale distortions on the NAD - MR 1972 datum (1000 km < chord < 2000 km)

Fig. 5-16

Differences After Transforming AUS to NWL9D
(in Metres) 3 Transformation Parameters
(NWL9D – AUS)

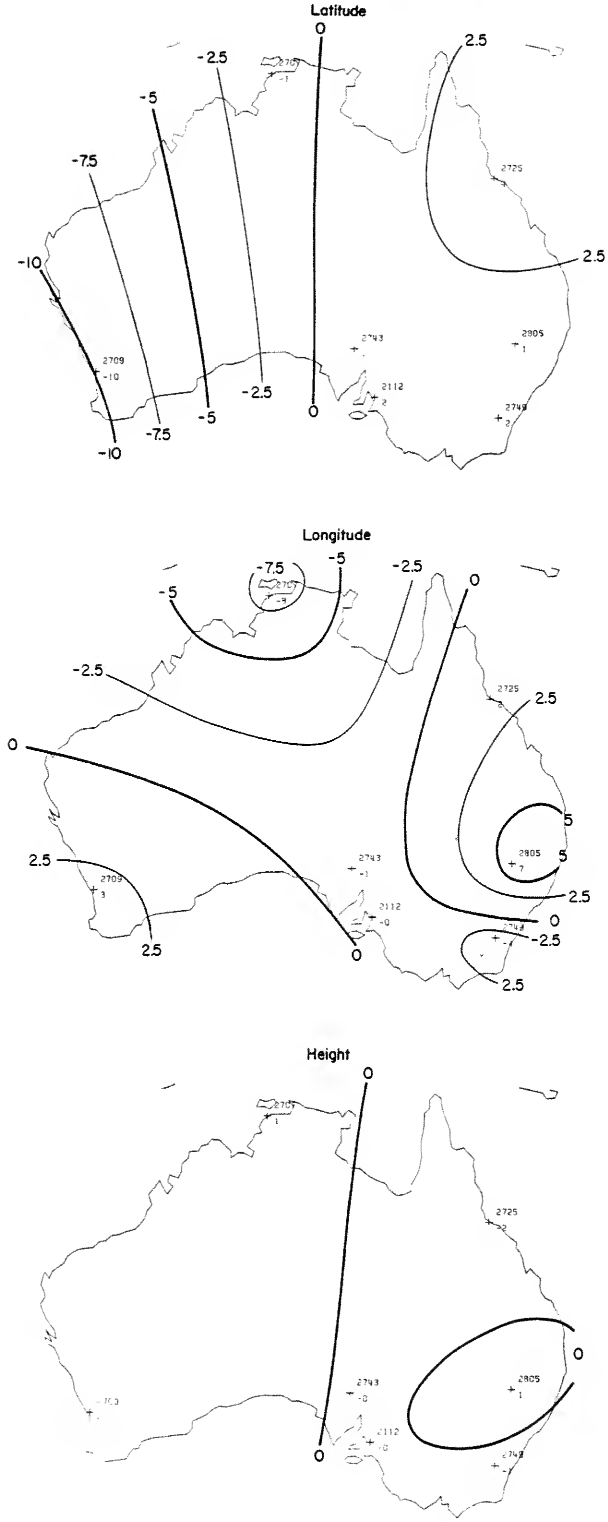
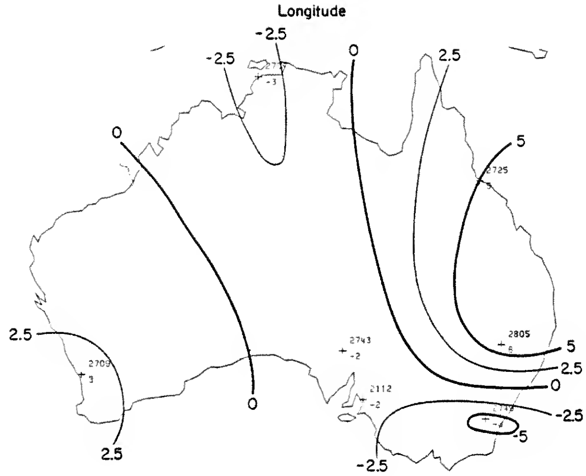


Fig. 5-17

Differences After Transforming AUS to NWL9D
(in Metres) 7 Transformation Parameters
(NWL9D - AUS)

Latitude

Longitude



Height

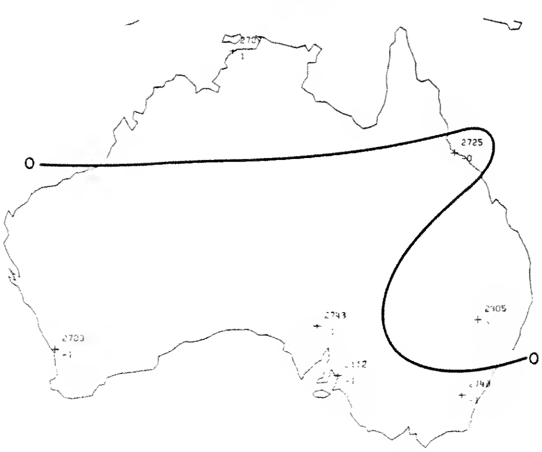


Fig. 5-18

Differences After Transforming SAD69 to NWL9D
(in Metres) 3 Transformation Parameters
(NWL9D-SAD69)

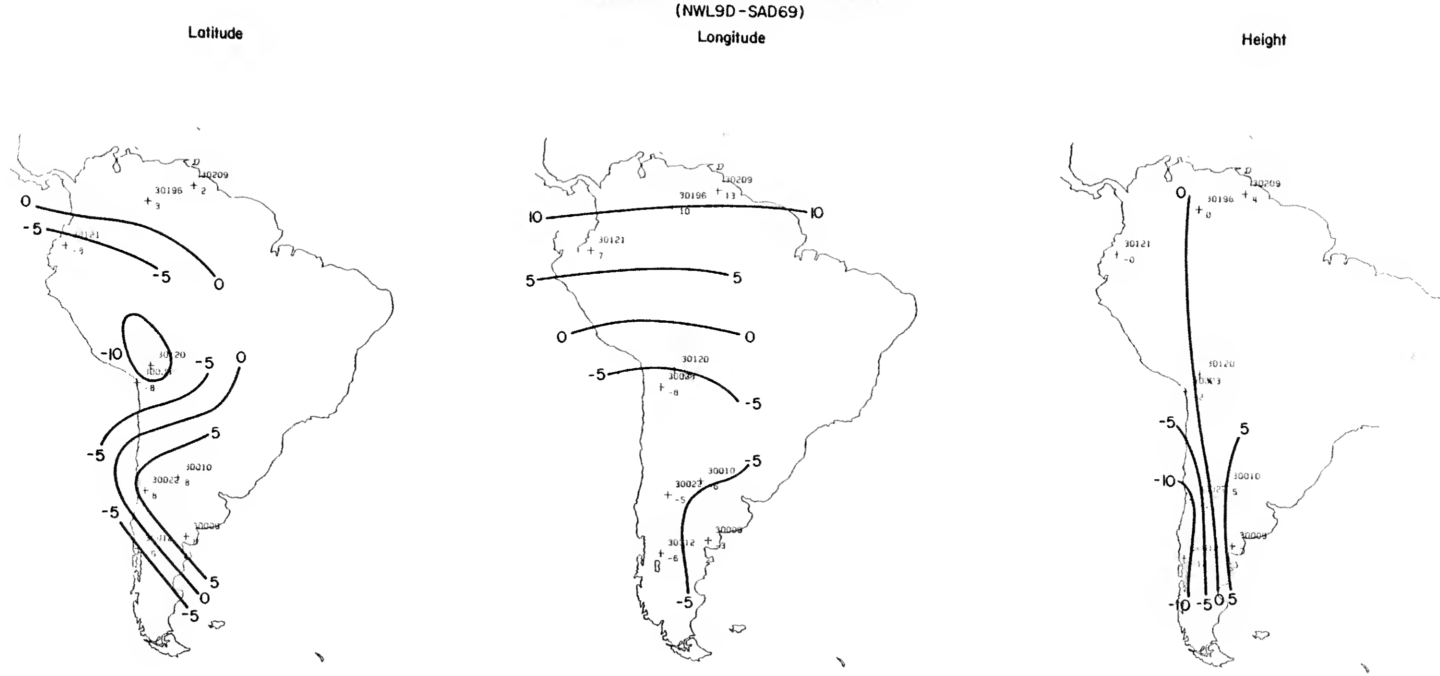


Fig. 5-19

Differences After Transforming SAD69 to NWL9D
(in Metres) 7 Transformation Parameters
(NWL9D - SAD69)

Latitude | Longitude | Height

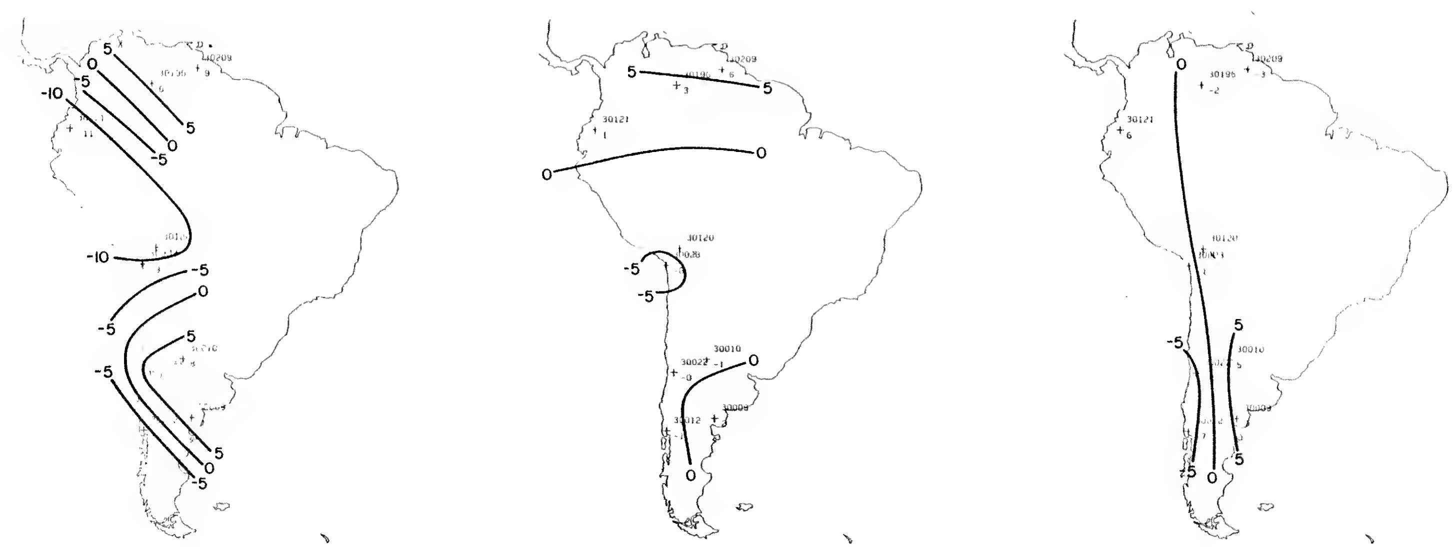


Table 5-3

Transformation

NAD-27 -TO- NWL-9D (3 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION PARAMETERS
(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS
-0.28110091D+02	0.15277281D+03	0.17850114D+03
± 0.39	± 0.39	± 0.39

VARIANCE - COVARIANCE MATRIX

MO2= 1.56

0.14993720D+00	0.0	0.0
0.0	0.14993720D+00	0.0
0.0	0.0	0.14993720D+00

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.0	0.0
0.0	0.10000000D+01	0.0
0.0	0.0	0.10000000D+01

Table 5-3 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD-27)				V2(NWL-9D)			V1 - V2		
10000	2.11	1.38	0.6	10000	-2.46	-1.61	-0.7	4.57	2.99	1.3
10003	-3.00	0.32	1.3	10003	1.86	-0.20	-0.8	-4.86	0.52	2.1
10006	-0.08	1.35	0.9	10006	0.05	-0.76	-0.5	-0.13	2.10	1.5
10018	-1.63	-0.50	1.5	10018	0.95	0.29	-0.9	-2.58	-0.80	2.4
10019	0.03	0.61	2.1	10019	-0.02	-0.30	-1.0	0.05	0.91	3.1
10020	-0.14	0.83	2.1	10020	0.07	-0.40	-1.0	-0.21	1.24	3.1
10021	-1.28	1.23	2.6	10021	0.63	-0.61	-1.3	-1.92	1.84	4.0
10022	-2.84	0.08	0.8	10022	1.52	-0.04	-0.4	-4.35	0.12	1.3
10023	-3.02	-0.35	1.5	10023	1.60	0.18	-0.8	-4.62	-0.53	2.2
10031	-0.16	-5.79	2.1	10031	0.04	1.64	-0.6	-0.20	-7.43	2.7
10055	6.95	-9.28	0.4	10055	-1.46	1.96	-0.1	8.41	-11.23	0.5
20003	0.72	-6.69	0.1	20003	-0.19	1.74	-0.0	0.91	-8.43	0.1
20016	-3.83	-0.67	2.6	20016	1.70	0.30	-1.2	-5.53	-0.97	3.8
30025	1.95	-0.11	1.5	30025	-0.67	0.04	-0.5	2.63	-0.15	2.0
30028	0.62	0.99	0.5	30028	-0.46	-0.74	-0.4	1.08	1.72	0.8
30029	8.25	-4.01	-0.1	30029	-1.98	0.96	0.0	10.23	-4.97	-0.1
30098	5.69	-7.45	-0.7	30098	-1.26	1.64	0.2	6.95	-9.09	-0.9
30099	9.37	3.29	-0.0	30099	-3.92	-1.37	0.0	13.29	4.66	-0.1
51008	0.84	0.75	0.4	51008	-0.20	-0.18	-0.1	1.04	0.93	0.4
51014	-4.09	2.29	0.5	51014	0.87	-0.49	-0.1	-4.96	2.78	0.6
51015	-1.21	2.60	0.9	51015	0.23	-0.50	-0.2	-1.44	3.10	1.1
51025	-2.11	-0.26	0.5	51025	1.15	0.14	-0.3	-3.26	-0.40	0.8
51030	-1.14	-0.19	-0.3	51030	2.28	0.39	0.6	-3.42	-0.58	-0.9
51033	-0.06	0.56	-0.0	51033	0.13	-1.19	0.0	-0.20	1.75	-0.0
51039	-2.89	-0.60	0.1	51039	1.68	0.35	-0.0	-4.56	-0.95	0.1
51041	0.60	0.57	0.1	51041	-1.27	-1.20	-0.1	1.87	1.77	0.2
51043	4.58	3.25	0.1	51043	-2.51	-1.78	-0.0	7.10	5.04	0.1
51044	2.49	1.55	-0.5	51044	-1.80	-1.12	0.4	4.28	2.67	-0.9
51048	-0.64	-0.95	-0.3	51048	0.45	0.66	0.2	-1.09	-1.61	-0.5
51052	-1.80	-0.88	-2.5	51052	0.67	0.33	0.9	-2.47	-1.21	-3.4
51066	6.57	-5.97	-0.9	51066	-1.51	1.37	0.2	8.08	-7.34	-1.1
51067	-2.01	-1.07	0.7	51067	1.83	0.97	-0.7	-3.85	-2.04	1.4
51068	-1.80	1.56	1.5	51068	0.47	-0.40	-0.4	-2.27	1.97	1.8
51069	-1.61	-1.05	1.1	51069	0.43	0.28	-0.3	-2.04	-1.32	1.4
51074	7.66	-3.76	-0.8	51074	-1.73	0.85	0.2	9.39	-4.61	-1.0
51089	5.15	-7.36	-1.3	51089	-1.14	1.63	0.3	6.29	-8.99	-1.6
51103	-3.11	-1.04	-0.5	51103	1.60	0.54	0.3	-4.71	-1.58	-0.8
51121	-3.86	0.60	0.7	51121	1.88	-0.29	-0.3	-5.73	0.89	1.0
51123	-3.27	-0.03	1.1	51123	1.68	0.01	-0.6	-4.96	-0.04	1.7
51124	-2.68	0.14	0.6	51124	1.69	-0.09	-0.4	-4.37	0.23	0.9
51125	-5.55	2.49	1.5	51125	1.96	-0.88	-0.5	-7.51	3.36	2.0
51126	-5.19	1.60	1.1	51126	1.50	-0.46	-0.3	-6.69	2.06	1.4
52001	3.56	-0.22	0.1	52001	-0.87	0.05	-0.0	4.44	-0.28	0.1
52063	5.22	-6.22	-0.3	52063	-1.22	1.45	0.1	6.44	-7.67	-0.4
10008	2.53	3.18	-0.5	10008	-1.48	-1.86	0.3	4.01	5.04	-0.8

Table 5-3 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD-27)				V2(NWL-9D)			V1 - V2		
10056	0.93	-7.63	-1.8	10056	-0.21	1.72	0.4	1.14	-9.36	-2.2
10071	-0.16	-1.44	0.8	10071	0.09	0.84	-0.4	-0.25	-2.28	1.2
20176	-1.32	-1.77	-1.9	20176	0.44	0.58	0.6	-1.76	-2.35	-2.5
20177	-2.86	-2.56	-2.0	20177	1.11	0.99	0.8	-3.96	-3.55	-2.8
20208	0.37	-4.30	0.6	20208	-0.13	1.49	-0.2	0.49	-5.78	0.8
30030	2.90	-0.20	0.8	30030	-1.63	0.11	-0.4	4.52	-0.31	1.2
51004	1.90	-0.67	-0.4	51004	-0.64	0.23	0.1	2.53	-0.89	-0.5
51005	-1.04	-0.51	0.8	51005	0.42	0.21	-0.3	-1.46	-0.72	1.1
51006	-4.73	-1.05	-0.8	51006	1.89	0.42	0.3	-6.63	-1.47	-1.1
51007	-1.84	1.13	-0.2	51007	0.58	-0.35	0.1	-2.42	1.49	-0.3
51009	-4.78	0.35	-0.0	51009	1.94	-0.14	0.0	-6.73	0.49	-0.0
51010	-3.58	0.40	0.5	51010	1.11	-0.12	-0.1	-4.69	0.53	0.6
51011	5.03	-5.16	0.7	51011	-1.33	1.36	-0.2	6.36	-6.52	0.9
51013	-3.55	2.90	1.2	51013	0.84	-0.68	-0.3	-4.39	3.58	1.5
51017	4.61	1.22	0.9	51017	-1.11	-0.29	-0.2	5.72	1.51	1.2
51019	6.00	1.36	0.4	51019	-1.25	-0.28	-0.1	7.25	1.64	0.4
51020	4.75	-1.29	2.3	51020	-0.95	0.26	-0.5	5.70	-1.56	2.8
51021	8.62	0.65	0.7	51021	-1.50	-0.11	-0.1	10.12	0.76	0.9
51022	6.75	-2.88	0.7	51022	-1.15	0.49	-0.1	7.91	-3.38	0.8
51023	4.76	-6.76	2.8	51023	-0.77	1.09	-0.4	5.52	-7.85	3.2
51024	-3.05	-0.53	0.6	51024	1.40	0.24	-0.3	-4.45	-0.77	0.8
51026	-2.53	0.26	0.8	51026	2.32	-0.23	-0.7	-4.85	0.49	1.6
51027	-0.29	1.11	-0.4	51027	0.39	-1.50	0.5	-0.68	2.61	-0.9
51028	-1.74	-0.17	0.7	51028	1.65	0.16	-0.6	-3.40	-0.33	1.3
51029	-1.11	0.31	-0.5	51029	2.77	-0.76	1.2	-3.88	1.07	-1.6
51031	-0.59	0.06	0.3	51031	0.39	-0.04	-0.2	-0.98	0.11	0.6
51032	0.53	0.81	-0.3	51032	-0.53	-0.81	0.3	1.06	1.62	-0.7
51056	3.62	-0.28	0.1	51056	-1.68	0.13	-0.1	5.30	-0.42	0.2
51057	4.39	-1.66	-0.4	51057	-1.54	0.58	0.1	5.93	-2.23	-0.5
51058	5.17	-4.69	-1.7	51058	-1.38	1.25	0.4	6.55	-5.94	-2.1
51095	7.10	-0.76	2.9	51095	-1.28	0.14	-0.5	8.38	-0.90	3.4

UNIT OF RESIDUALS (METERS)

Table 5-4

Transformation

NAD 27 -TO- NWL-9D (4 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION AND 1 SCALE PARAMETERS

(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DL (X1.D+6)
-0.26914824D+02	0.16328066D+03	0.17043654D+03	0.21273913D+01
± 0.38	± 1.44	± 1.13	± 0.28

VARIANCE - COVARIANCE MATRIX

MO2= 1.25

0.14533139D+00	0.22128622D+00	-0.16983357D+00	0.44801010D-07
0.22128622D+00	0.20655349D+01	-0.14930435D+01	0.39385534D-06
-0.16983357D+00	-0.14930435D+01	0.12660467D+01	-0.30227756D-06
0.44801010D-07	0.39385534D-06	-0.30227756D-06	0.79738888D-13

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.40388561D+00	-0.39593051D+00	0.41617214D+00
0.40388561D+00	0.10000000D+01	-0.92327555D+00	0.97047728D+00
-0.39593051D+00	-0.92327555D+00	0.10000000D+01	-0.95136235D+00
0.41617214D+00	0.97047728D+00	-0.95136235D+00	0.10000000D+01

Table 5-4 (cont'd)

RESIDUALS V
SPHERICAL

	V1 (NAD 27)				V2 (NWL-9D)			V1 - V2		
10000	1.76	2.09	0.4	10000	-2.05	-2.44	-0.5	3.80	4.53	0.9
10003	-2.41	-0.30	1.1	10003	1.50	0.19	-0.7	-3.90	-0.49	1.8
10006	-0.33	1.59	0.8	10006	0.19	-0.89	-0.4	-0.52	2.48	1.2
10018	-0.60	-0.33	1.3	10018	0.35	0.19	-0.7	-0.95	-0.52	2.0
10019	-0.31	-0.70	1.8	10019	0.15	0.35	-0.9	-0.46	-1.05	2.7
10020	-0.23	-0.51	1.8	10020	0.11	0.25	-0.9	-0.34	-0.75	2.7
10021	-1.22	-0.05	2.4	10021	0.60	0.02	-1.2	-1.82	-0.07	3.6
10022	-2.39	-0.91	0.6	10022	1.27	0.49	-0.3	-3.66	-1.40	0.9
10023	-2.39	-1.23	1.2	10023	1.26	0.65	-0.6	-3.65	-1.88	1.9
10031	0.52	-2.92	1.5	10031	-0.15	0.83	-0.4	0.67	-3.75	1.9
10055	7.49	-5.45	-0.5	10055	-1.58	1.15	0.1	9.07	-6.60	-0.6
20003	1.62	-3.66	-0.6	20003	-0.42	0.95	0.2	2.04	-4.61	-0.8
20016	-2.80	-1.54	2.3	20016	1.24	0.68	-1.0	-4.04	-2.22	3.4
30025	1.68	-2.11	1.1	30025	-0.58	0.73	-0.4	2.25	-2.84	1.5
30028	0.21	0.23	0.3	30028	-0.16	-0.17	-0.2	0.37	0.40	0.5
30029	6.94	-0.70	-0.9	30029	-1.67	0.17	0.2	8.61	-0.87	-1.2
30098	5.81	-3.69	-1.6	30098	-1.28	0.81	0.3	7.10	-4.51	-1.9
30099	7.82	4.86	-0.5	30099	-3.27	-2.03	0.2	11.09	6.89	-0.7
51008	1.73	-1.94	-0.2	51008	-0.41	0.46	0.1	2.15	-2.40	-0.3
51014	-2.12	-0.10	-0.1	51014	0.45	0.02	0.0	-2.57	-0.13	-0.2
51015	1.19	0.13	0.2	51015	-0.23	-0.02	-0.0	1.42	0.15	0.2
51025	-1.12	-0.61	0.3	51025	0.61	0.33	-0.2	-1.73	-0.94	0.4
51030	-1.00	-0.10	-0.4	51030	2.01	0.20	0.8	-3.01	-0.31	-1.1
51033	-0.37	0.51	-0.1	51033	0.79	-1.08	0.2	-1.17	1.59	-0.3
51039	-1.90	0.03	-0.2	51039	1.10	-0.02	0.1	-3.00	0.05	-0.3
51041	0.31	0.88	-0.0	51041	-0.64	-1.84	0.1	0.95	2.71	-0.1
51043	3.60	4.66	-0.3	51043	-1.97	-2.55	0.2	5.57	7.21	-0.4
51044	2.01	2.75	-0.8	51044	-1.46	-1.99	0.6	3.47	4.75	-1.3
51048	-0.24	0.13	-0.5	51048	0.17	-0.09	0.4	-0.41	0.22	-0.9
51052	-0.85	1.12	-2.9	51052	0.31	-0.41	1.1	-1.17	1.53	-4.0
51066	5.83	-2.36	-1.7	51066	-1.34	0.54	0.4	7.17	-2.91	-2.1
51067	-1.46	-0.91	0.6	51067	1.33	0.83	-0.5	-2.79	-1.75	1.1
51068	-0.41	-0.58	0.9	51068	0.10	0.15	-0.2	-0.51	-0.73	1.2
51069	-0.57	-3.34	0.6	51069	0.15	0.88	-0.2	-0.72	-4.23	0.7
51074	7.34	-0.07	-1.7	51074	-1.66	0.02	0.4	9.00	-0.09	-2.0
51089	5.54	-3.67	-2.2	51089	-1.23	0.81	0.5	6.77	-4.48	-2.6
51103	-2.22	0.17	-0.8	51103	1.14	-0.09	0.4	-3.37	0.26	-1.2
51121	-2.77	0.07	0.4	51121	1.35	-0.03	-0.2	-4.13	0.10	0.6
51123	-2.20	0.89	0.8	51123	1.13	-0.46	-0.4	-3.33	1.34	1.2
51124	-1.76	0.55	0.3	51124	1.11	-0.35	-0.2	-2.86	0.89	0.5
51125	-4.32	1.19	1.2	51125	1.52	-0.42	-0.4	-5.84	1.61	1.6
51126	-3.84	-0.20	0.7	51126	1.11	0.06	-0.2	-4.94	-0.26	0.9
52001	3.59	-3.08	-0.5	52001	-0.88	0.75	0.1	4.47	-3.83	-0.7
52063	5.78	-2.72	-1.1	52063	-1.35	0.63	0.3	7.13	-3.35	-1.4
10008	1.01	3.29	-0.8	10008	-0.59	-1.92	0.5	1.60	5.21	-1.3

Table 5-4 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD 27)				V2(NWL-9D)			V1 - V2		
10056	2.11	-4.26	-2.6	10056	-0.48	0.96	0.6	2.59	-5.22	-3.2
10071	0.23	-0.04	0.5	10071	-0.13	0.02	-0.3	0.36	-0.06	0.7
20176	-0.27	0.47	-2.4	20176	0.09	-0.16	0.8	-0.36	0.63	-3.2
20177	-1.73	-0.84	-2.4	20177	0.67	0.32	0.9	-2.40	-1.16	-3.4
20208	0.98	-1.93	0.1	20208	-0.34	0.67	-0.0	1.31	-2.59	0.1
30030	2.80	1.40	0.5	30030	-1.57	-0.79	-0.3	4.37	2.19	0.7
51004	1.91	-2.70	-0.8	51004	-0.64	0.91	0.3	2.56	-3.61	-1.1
51005	-0.85	-2.13	0.5	51005	0.34	0.86	-0.2	-1.19	-2.99	0.7
51006	-4.25	-2.57	-1.1	51006	1.70	1.03	0.4	-5.95	-3.60	-1.5
51007	-1.18	-0.88	-0.6	51007	0.37	0.27	0.2	-1.55	-1.15	-0.9
51009	-3.99	-0.96	-0.3	51009	1.63	0.39	0.1	-5.62	-1.35	-0.5
51010	-2.67	-1.49	0.0	51010	0.83	0.47	-0.0	-3.50	-1.96	0.0
51011	5.32	-7.75	0.2	51011	-1.40	2.05	-0.0	6.72	-9.79	0.2
51013	-1.64	0.86	0.6	51013	0.39	-0.20	-0.2	-2.03	1.06	0.8
51017	4.30	-1.72	0.3	51017	-1.03	0.41	-0.1	5.34	-2.13	0.4
51019	5.55	-2.02	-0.4	51019	-1.16	0.42	0.1	6.71	-2.44	-0.5
51020	3.83	-4.76	1.5	51020	-0.77	0.96	-0.3	4.60	-5.72	1.8
51021	8.40	-3.37	-0.2	51021	-1.46	0.59	0.0	9.86	-3.96	-0.3
51022	6.04	-6.97	-0.3	51022	-1.03	1.19	0.1	7.07	-8.16	-0.4
51023	3.61	-11.03	1.7	51023	-0.58	1.77	-0.3	4.19	-12.80	1.9
51024	-1.71	-0.67	0.3	51024	0.78	0.31	-0.1	-2.49	-0.98	0.5
51026	-2.06	0.11	0.7	51026	1.89	-0.10	-0.6	-3.95	0.22	1.3
51027	-0.16	0.92	-0.5	51027	0.21	-1.25	0.7	-0.37	2.17	-1.2
51028	-1.62	-0.63	0.5	51028	1.54	0.60	-0.5	-3.16	-1.23	1.0
51029	-1.22	0.22	-0.5	51029	3.06	-0.54	1.3	-4.29	0.75	-1.9
51031	-0.71	-0.84	0.1	51031	0.47	0.55	-0.1	-1.18	-1.39	0.2
51032	0.30	0.31	-0.5	51032	-0.30	-0.31	0.5	0.60	0.61	-1.0
51056	3.30	1.64	-0.3	51056	-1.53	-0.76	0.1	4.83	2.41	-0.4
51057	4.17	0.86	-0.9	51057	-1.46	-0.30	0.3	5.62	1.16	-1.2
51058	5.16	-1.50	-2.4	51058	-1.38	0.40	0.6	6.53	-1.89	-3.0
51095	6.60	-4.66	2.0	51095	-1.19	0.84	-0.4	7.78	-5.50	2.3

UNIT OF RESIDUALS (METERS)

Table 5-5

Transformation
NAD-27 -TO- NWL-9D (BURSA)
****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-21.06	166.19	175.10	2.13	0.29	-0.07	-0.20
± 1.94	± 2.56	± 3.08	± 0.27	± 0.06	± 0.06	± 0.12

VARIANCE - COVARIANCE MATRIX

SO2= 1.11

0.378D+01	0.819D+00	0.120D+01	0.398D-07	0.445D-06	0.383D-06	-0.230D-06
0.819D+00	0.655D+01	0.492D+01	0.350D-06	0.187D-08	0.162D-06	-0.125D-05
0.120D+01	0.492D+01	0.946D+01	-0.268D-06	0.684D-07	0.268D-06	-0.166D-05
0.398D-07	0.350D-06	-0.268D-06	0.708D-13	0.625D-29	0.526D-29	-0.535D-28
0.445D-06	0.187D-08	0.684D-07	0.136D-28	0.817D-13	0.109D-13	-0.126D-13
0.383D-06	0.162D-06	0.268D-06	0.342D-28	0.109D-13	0.868D-13	-0.443D-13
-0.230D-06	-0.125D-05	-0.166D-05	-0.693D-28	-0.126D-13	-0.443D-13	0.330D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.165D+00	0.201D+00	0.769D-01	0.801D+00	0.669D+00	-0.206D+00
0.165D+00	0.100D+01	0.625D+00	0.513D+00	0.256D-02	0.215D+00	-0.846D+00
0.201D+00	0.625D+00	0.100D+01	-0.328D+00	0.778D-01	0.296D+00	-0.937D+00
0.769D-01	0.513D+00	-0.328D+00	0.100D+01	0.822D-16	0.672D-16	-0.350D-15
0.801D+00	0.256D-02	0.778D-01	0.822D-16	0.100D+01	0.130D+00	-0.767D-01
0.669D+00	0.215D+00	0.296D+00	0.672D-16	0.130D+00	0.100D+01	-0.262D+00
-0.206D+00	-0.846D+00	-0.937D+00	-0.350D-15	-0.767D-01	-0.262D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-5 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD-27)				V2(NWL-9D)			V1 - V2		
10000	1.37	1.92	0.5	10000	-1.60	-2.24	-0.6	2.97	4.17	1.1
10003	-2.00	0.09	1.1	10003	1.24	-0.06	-0.7	-3.24	0.15	1.7
10006	-0.40	1.51	0.7	10006	0.23	-0.85	-0.4	-0.63	2.35	1.2
10018	-0.59	0.32	1.8	10018	0.34	-0.18	-1.1	-0.94	0.50	2.9
10019	0.50	-0.76	1.1	10019	-0.25	0.38	-0.5	0.75	-1.13	1.6
10020	0.58	-0.43	1.1	10020	-0.28	0.21	-0.6	0.86	-0.64	1.7
10021	-0.45	0.11	1.8	10021	0.22	-0.05	-0.9	-0.68	0.16	2.7
10022	-1.79	-0.58	0.3	10022	0.96	0.31	-0.2	-2.74	-0.88	0.5
10023	-1.85	-0.81	1.1	10023	0.98	0.43	-0.6	-2.83	-1.23	1.7
10031	-1.19	-2.25	2.7	10031	0.34	0.64	-0.8	-1.52	-2.88	3.5
10055	5.07	-4.79	0.8	10055	-1.07	1.01	-0.2	6.14	-5.79	1.0
20003	-0.17	-2.81	0.8	20003	0.04	0.73	-0.2	-0.21	-3.55	1.0
20016	-2.26	-0.91	2.4	20016	1.00	0.40	-1.1	-3.26	-1.31	3.4
30025	2.86	-2.07	0.0	30025	-0.99	0.71	-0.0	3.84	-2.78	0.0
30028	0.71	0.08	-0.2	30028	-0.53	-0.06	0.2	1.25	0.15	-0.4
30029	4.77	-1.50	-0.7	30029	-1.15	0.36	0.2	5.92	-1.86	-0.9
30098	3.40	-3.36	-0.5	30098	-0.75	0.74	0.1	4.15	-4.10	-0.6
30099	6.91	3.89	-0.7	30099	-2.89	-1.63	0.3	9.80	5.52	-1.0
51008	3.14	-1.26	-1.2	51008	-0.75	0.30	0.3	3.89	-1.56	-1.5
51014	-0.90	1.01	-0.5	51014	0.19	-0.22	0.1	-1.09	1.23	-0.7
51015	2.43	1.42	-0.1	51015	-0.47	-0.27	0.0	2.90	1.69	-0.1
51025	-0.84	-0.00	0.6	51025	0.46	0.00	-0.3	-1.30	-0.00	0.9
51030	-1.02	0.01	-0.3	51030	2.04	-0.02	0.6	-3.06	0.03	-0.9
51033	-0.30	0.36	-0.3	51033	0.64	-0.77	0.5	-0.94	1.13	-0.8
51039	-2.15	0.69	0.5	51039	1.25	-0.40	-0.3	-3.40	1.09	0.8
51041	0.16	0.72	-0.1	51041	-0.33	-1.52	0.1	0.49	2.25	-0.2
51043	2.78	4.08	-0.3	51043	-1.53	-2.24	0.2	4.30	6.32	-0.4
51044	1.32	2.52	-0.6	51044	-0.96	-1.83	0.4	2.28	4.35	-1.0
51048	-0.80	0.47	0.0	51048	0.56	-0.33	-0.0	-1.36	0.80	0.1
51052	-1.93	1.89	-1.8	51052	0.71	-0.70	0.7	-2.65	2.59	-2.4
51066	3.46	-2.72	-1.2	51066	-0.79	0.62	0.3	4.25	-3.34	-1.4
51067	-1.48	-0.54	0.9	51067	1.34	0.49	-0.8	-2.82	-1.04	1.7
51068	0.72	0.27	0.5	51068	-0.19	-0.07	-0.1	0.91	0.35	0.6
51069	0.65	-2.63	-0.1	51069	-0.17	0.70	0.0	0.82	-3.33	-0.2
51074	4.93	-0.08	-0.8	51074	-1.12	0.02	0.2	6.05	-0.10	-1.0
51089	3.20	-3.13	-0.9	51089	-0.71	0.70	0.2	3.91	-3.83	-1.1
51103	-2.82	0.82	0.0	51103	1.45	-0.42	-0.0	-4.27	1.24	0.1
51121	-2.40	0.71	0.6	51121	1.17	-0.35	-0.3	-3.57	1.06	0.9
51123	-2.60	1.62	1.7	51123	1.34	-0.83	-0.9	-3.93	2.45	2.5
51124	-1.88	1.15	0.9	51124	1.19	-0.73	-0.6	-3.07	1.88	1.5
51125	-3.57	1.93	1.1	51125	1.26	-0.68	-0.4	-4.83	2.61	1.4
51126	-2.85	0.61	0.4	51126	0.82	-0.18	-0.1	-3.68	0.79	0.5
52001	5.18	-2.77	-1.9	52001	-1.27	0.68	0.5	6.45	-3.45	-2.4
52063	3.61	-2.07	0.2	52063	-0.84	0.48	-0.0	4.46	-2.56	0.2
10008	1.08	2.41	-1.4	10008	-0.63	-1.41	0.8	1.70	3.81	-2.3

Table 5-5 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD-27)				V2(NWL-9D)			V1 - V2		
10056	0.13	-3.17	-1.0	10056	-0.03	0.72	0.2	0.15	-3.89	-1.3
10071	-0.54	0.31	1.1	10071	0.31	-0.18	-0.7	-0.85	0.49	1.8
20176	-1.49	1.33	-1.1	20176	0.49	-0.44	0.4	-1.98	1.77	-1.5
20177	-2.61	0.01	-1.3	20177	1.01	-0.00	0.5	-3.62	0.02	-1.8
20208	-0.39	-1.35	1.2	20208	0.14	0.47	-0.4	-0.53	-1.82	1.6
30030	1.87	1.44	1.0	30030	-1.05	-0.81	-0.5	2.93	2.25	1.5
51004	3.08	-2.51	-1.8	51004	-1.04	0.84	0.6	4.12	-3.35	-2.4
51005	0.09	-1.88	-0.2	51005	-0.04	0.76	0.1	0.13	-2.64	-0.3
51006	-3.38	-2.19	-1.6	51006	1.35	0.88	0.6	-4.73	-3.06	-2.2
51007	-0.08	-0.37	-1.3	51007	0.02	0.12	0.4	-0.10	-0.48	-1.8
51009	-3.24	-0.44	-0.6	51009	1.32	0.18	0.2	-4.56	-0.62	-0.8
51010	-1.63	-0.88	-0.5	51010	0.51	0.27	0.2	-2.14	-1.15	-0.7
51011	6.74	-7.35	-1.0	51011	-1.78	1.94	0.3	8.52	-9.30	-1.2
51013	-0.57	1.93	0.4	51013	0.13	-0.45	-0.1	-0.70	2.38	0.5
51017	5.98	-1.56	-1.2	51017	-1.44	0.38	0.3	7.41	-1.94	-1.5
51019	7.48	-1.86	-2.2	51019	-1.56	0.39	0.5	9.04	-2.25	-2.7
51020	5.89	-4.83	-0.6	51020	-1.18	0.97	0.1	7.08	-5.80	-0.7
51021	10.61	-2.98	-2.4	51021	-1.84	0.52	0.4	12.46	-3.50	-2.8
51022	8.39	-6.81	-2.7	51022	-1.43	1.16	0.5	9.83	-7.97	-3.1
51023	6.16	-11.04	-0.9	51023	-0.99	1.78	0.2	7.16	-12.82	-1.1
51024	-1.51	0.13	0.8	51024	0.69	-0.06	-0.4	-2.21	0.18	1.2
51026	-1.92	0.43	0.8	51026	1.76	-0.39	-0.7	-3.68	0.83	1.6
51027	-0.00	1.04	-0.5	51027	0.01	-1.41	0.7	-0.01	2.46	-1.2
51028	-1.31	-0.50	0.4	51028	1.24	0.48	-0.4	-2.55	-0.98	0.7
51029	-1.14	0.18	-0.6	51029	2.85	-0.45	1.6	-3.99	0.63	-2.2
51031	-0.14	-0.81	-0.3	51031	0.09	0.54	0.2	-0.23	-1.34	-0.5
51032	0.65	0.24	-0.8	51032	-0.65	-0.24	0.8	1.30	0.48	-1.6
51056	2.14	1.54	0.2	51056	-0.99	-0.72	-0.1	3.14	2.26	0.3
51057	2.62	0.86	-0.2	51057	-0.92	-0.30	0.1	3.53	1.16	-0.3
51058	3.14	-1.30	-1.4	51058	-0.84	0.35	0.4	3.98	-1.65	-1.8
51095	8.81	-4.43	-0.2	51095	-1.58	0.80	0.0	10.39	-5.23	-0.3

UNIT OF RESIDUALS (METERS)

Table 5-6

Transformation

NAD-27 -TO- NWL-9D (MOL-MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-28.34	152.90	179.07	2.13	0.29	-0.07	-0.20
± 0.34	± 0.35	± 0.34	± 0.27	± 0.06	± 0.06	± 0.12

VARIANCE - COVARIANCE MATRIX

SO2= 1.11

0.113D+00	0.170D-02	0.956D-04	-0.122D-07	0.138D-08	-0.186D-07	0.919D-08
0.170D-02	0.124D+00	-0.119D-02	0.329D-08	0.114D-07	-0.788D-08	0.707D-07
0.956D-04	-0.119D-02	0.113D+00	0.156D-07	-0.131D-08	-0.130D-07	-0.767D-08
-0.122D-07	0.329D-08	0.156D-07	0.708D-13	-0.439D-30	-0.226D-30	0.493D-30
0.138D-08	0.114D-07	-0.131D-08	-0.219D-30	0.817D-13	0.109D-13	-0.126D-13
-0.186D-07	-0.788D-08	-0.130D-07	-0.651D-31	0.109D-13	0.868D-13	-0.443D-13
0.919D-08	0.707D-07	-0.767D-08	0.175D-29	-0.126D-13	-0.443D-13	0.330D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.143D-01	0.848D-03	-0.137D+00	0.144D-01	-0.188D+00	0.476D-01
0.143D-01	0.100D+01	-0.101D-01	0.350D-01	0.113D+00	-0.759D-01	0.349D+00
0.848D-03	-0.101D-01	0.100D+01	0.175D+00	-0.136D-01	-0.131D+00	-0.397D-01
-0.137D+00	0.350D-01	0.175D+00	0.100D+01	-0.577D-17	-0.289D-17	0.323D-17
0.144D-01	0.113D+00	-0.136D-01	-0.577D-17	0.100D+01	0.130D+00	-0.767D-01
-0.188D+00	-0.759D-01	-0.131D+00	-0.289D-17	0.130D+00	0.100D+01	-0.262D+00
0.476D-01	0.349D+00	-0.397D-01	0.323D-17	-0.767D-01	-0.262D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-7

Transformation

NAD-27 -TO- NWL-9D (VEIS MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	ALPHA SECONDS	KSI SECONDS	ETA SECONDS
-28.34	152.90	179.07	2.13	0.26	-0.19	-0.17
± 0.34	± 0.35	± 0.34	± 0.27	± 0.06	± 0.12	± 0.06

VARIANCE - COVARIANCE MATRIX

SO2= 1.11

0.113D+00	0.170D-02	0.956D-04	-0.122D-07	0.141D-07	0.119D-07	0.972D-08
0.170D-02	0.124D+00	-0.119D-02	0.329D-08	0.509D-08	0.711D-07	-0.105D-07
0.956D-04	-0.119D-02	0.113D+00	0.156D-07	0.998D-08	-0.566D-08	0.984D-08
-0.122D-07	0.329D-08	0.156D-07	0.708D-13	-0.377D-30	-0.932D-30	-0.164D-29
0.141D-07	0.509D-08	0.998D-08	-0.593D-30	0.714D-13	-0.381D-14	0.499D-15
0.119D-07	0.711D-07	-0.566D-08	0.548D-31	-0.381D-14	0.338D-12	0.151D-13
0.972D-08	-0.105D-07	0.984D-08	-0.241D-29	0.499D-15	0.151D-13	0.895D-13

COEFFICIENTS OF CORRELATION

0.100D+01	0.143D-01	0.848D-03	-0.137D+00	0.157D+00	0.607D-01	0.967D-01
0.143D-01	0.100D+01	-0.101D-01	0.350D-01	0.540D-01	0.347D+00	-0.997D-01
0.848D-03	-0.101D-01	0.100D+01	0.175D+00	0.111D+00	-0.290D-01	0.980D-01
-0.137D+00	0.350D-01	0.175D+00	0.100D+01	-0.530D-17	-0.603D-17	-0.207D-16
0.157D+00	0.540D-01	0.111D+00	-0.530D-17	0.100D+01	-0.245D-01	0.625D-02
0.607D-01	0.347D+00	-0.290D-01	-0.603D-17	-0.245D-01	0.100D+01	0.868D-01
0.967D-01	-0.997D-01	0.980D-01	-0.207D-16	0.625D-02	0.868D-01	0.100D+01

Table 5-8

Transformation

NAD 27 E -TO- NWL-9D (3 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION PARAMETERS
(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS
-0.27865631D+02	0.15115336D+03	0.17753369D+03
± 0.39	± 0.39	± 0.39

VARIANCE - COVARIANCE MATRIX

MO2= 0.95

0.15058588D+00	0.0	0.0
0.0	0.15058588D+00	0.0
0.0	0.0	0.15058588D+00

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.0	0.0
0.0	0.10000000D+01	0.0
0.0	0.0	0.10000000D+01

Table 5-8 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD 27 E)				V2(NWL-9D)			V1 - V2		
10003	-1.95	0.15	0.8	10003	1.21	-0.10	-0.5	-3.17	0.25	1.3
10018	-0.60	-0.80	1.0	10018	0.35	0.47	-0.6	-0.95	-1.27	1.5
10019	1.23	0.52	1.7	10019	-0.61	-0.26	-0.8	1.85	0.79	2.5
10020	1.05	0.75	1.6	10020	-0.51	-0.36	-0.8	1.56	1.12	2.4
10021	-0.10	1.14	2.2	10021	0.05	-0.57	-1.1	-0.16	1.71	3.2
10022	-1.72	-0.05	0.3	10022	0.92	0.03	-0.2	-2.63	-0.07	0.5
10023	-1.91	-0.49	0.9	10023	1.01	0.26	-0.5	-2.91	-0.75	1.4
20016	-2.68	-0.84	2.0	20016	1.19	0.37	-0.9	-3.87	-1.21	2.9
30025	3.28	-0.12	1.0	30025	-1.14	0.04	-0.3	4.42	-0.16	1.4
30028	1.64	0.86	0.1	30028	-1.23	-0.64	-0.1	2.87	1.50	0.2
51008	2.22	0.82	-0.3	51008	-0.53	-0.20	0.1	2.76	1.02	-0.4
51014	-2.75	2.31	-0.3	51014	0.59	-0.49	0.1	-3.34	2.80	-0.4
51015	0.12	2.63	0.0	51015	-0.02	-0.50	-0.0	0.14	3.13	0.0
51025	-1.04	-0.49	-0.1	51025	0.57	0.26	0.0	-1.61	-0.75	-0.1
51030	-0.57	-0.35	-0.5	51030	1.15	0.70	1.1	-1.72	-1.05	-1.6
51033	0.51	0.43	-0.2	51033	-1.07	-0.92	0.4	1.58	1.35	-0.5
51067	-1.14	-1.31	0.3	51067	1.04	1.20	-0.3	-2.18	-2.51	0.6
51068	-0.48	1.56	0.7	51068	0.12	-0.40	-0.2	-0.60	1.96	0.9
51069	-0.27	-1.03	0.4	51069	0.07	0.27	-0.1	-0.34	-1.30	0.5
51121	-2.74	0.40	0.1	51121	1.34	-0.19	-0.0	-4.08	0.59	0.1
51125	-4.32	2.37	0.8	51125	1.52	-0.83	-0.3	-5.85	3.21	1.1
51126	-3.90	1.55	0.4	51126	1.12	-0.45	-0.1	-5.02	2.00	0.5
52001	4.99	-0.12	-0.5	52001	-1.22	0.03	0.1	6.22	-0.14	-0.6
51004	3.23	-0.67	-0.9	51004	-1.08	0.23	0.3	4.31	-0.90	-1.2
51005	0.21	-0.57	0.3	51005	-0.08	0.23	-0.1	0.29	-0.80	0.4
51006	-3.49	-1.12	-1.3	51006	1.40	0.45	0.5	-4.89	-1.57	-1.9
51007	-0.53	1.12	-0.8	51007	0.17	-0.35	0.3	-0.69	1.47	-1.1
51009	-3.57	0.24	-0.6	51009	1.45	-0.10	0.3	-5.02	0.34	-0.9
51010	-2.28	0.37	-0.2	51010	0.71	-0.11	0.1	-2.99	0.48	-0.3
51011	6.42	-5.09	0.1	51011	-1.70	1.34	-0.0	8.12	-6.43	0.2
51013	-2.24	2.87	0.4	51013	0.53	-0.68	-0.1	-2.76	3.55	0.5
51017	6.06	1.34	0.5	51017	-1.46	-0.32	-0.1	7.52	1.66	0.6
51019	7.49	1.54	-0.1	51019	-1.57	-0.32	0.0	9.06	1.86	-0.1
51020	6.27	-1.10	1.9	51020	-1.26	0.22	-0.4	7.53	-1.32	2.3
51021	10.15	0.93	0.3	51021	-1.76	-0.16	-0.0	11.91	1.09	0.3
51022	8.30	-2.60	0.3	51022	-1.42	0.44	-0.0	9.72	-3.04	0.3
51023	6.33	-6.44	2.4	51023	-1.02	1.04	-0.4	7.35	-7.48	2.8
51024	-1.94	-0.80	-0.0	51024	0.89	0.37	0.0	-2.83	-1.16	-0.1
51026	-1.65	0.06	0.4	51026	1.51	-0.05	-0.4	-3.16	0.11	0.8
51027	0.44	0.95	-0.7	51027	-0.60	-1.29	0.9	1.04	2.25	-1.6
51028	-0.85	-0.32	0.3	51028	0.81	0.30	-0.3	-1.66	-0.62	0.6
51029	-0.61	0.20	-0.6	51029	1.52	-0.49	1.6	-2.12	0.69	-2.2
51031	0.47	-0.06	-0.1	51031	-0.31	0.04	0.0	0.79	-0.09	-0.1
51032	1.42	0.67	-0.7	51032	-1.42	-0.67	0.7	2.84	1.34	-1.3
51095	8.64	-0.50	2.4	51095	-1.55	0.09	-0.4	10.19	-0.59	2.9

UNIT OF RESIDUALS (METERS)

Table 5-9

Transformation

NAD 27 E -TO- NWL-9D (4 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION AND 1 SCALE PARAMETERS

(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)
-0.28148918D+02	0.16053536D+03	0.17071026D+03	0.18362904D+01
± 0.37	± 2.35	± 1.73	± 0.45

VARIANCE - COVARIANCE MATRIX

MO2= 0.85

0.13982613D+00	-0.16271198D+00	0.11833867D+00	-0.31846781D-07
-0.16271198D+00	0.55236613D+01	-0.39191787D+01	0.10547121D-05
0.11833867D+00	-0.39191787D+01	0.29852897D+01	-0.76708077D-06
-0.31846781D-07	0.10547121D-05	-0.76708077D-06	0.20643339D-12

COEFFICIENTS OF CORRELATION

0.10000000D+01	-0.18514493D+00	0.18316365D+00	-0.18744826D+00
-0.18514493D+00	0.10000000D+01	-0.96513554D+00	0.98771222D+00
0.18316365D+00	-0.96513554D+00	0.10000000D+01	-0.97714245D+00
-0.18744826D+00	0.98771222D+00	-0.97714245D+00	0.10000000D+01

Table 5-9 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD 27 E)				V2(NWL-9D)			V1 - V2		
10003	-1.61	0.43	0.7	10003	1.00	-0.27	-0.4	-2.61	0.70	1.2
10018	0.17	0.20	0.8	10018	-0.10	-0.12	-0.5	0.28	0.32	1.2
10019	0.69	0.26	1.6	10019	-0.34	-0.13	-0.8	1.04	0.38	2.4
10020	0.73	0.46	1.5	10020	-0.35	-0.22	-0.7	1.09	0.69	2.3
10021	-0.28	0.90	2.1	10021	0.14	-0.45	-1.0	-0.43	1.35	3.1
10022	-1.53	-0.05	0.3	10022	0.82	0.03	-0.1	-2.35	-0.08	0.4
10023	-1.56	-0.40	0.9	10023	0.82	0.21	-0.4	-2.38	-0.60	1.3
20016	-1.98	-0.67	1.9	20016	0.88	0.30	-0.8	-2.86	-0.97	2.7
30025	2.73	-0.91	0.9	30025	-0.94	0.32	-0.3	3.67	-1.23	1.2
30028	1.11	0.95	0.1	30028	-0.83	-0.71	-0.0	1.94	1.67	0.1
51008	2.64	-0.51	-0.5	51008	-0.63	0.12	0.1	3.28	-0.63	-0.6
51014	-1.35	1.27	-0.5	51014	0.29	-0.27	0.1	-1.64	1.54	-0.7
51015	1.89	1.54	-0.3	51015	-0.36	-0.29	0.1	2.25	1.83	-0.3
51025	-0.34	0.08	-0.2	51025	0.18	-0.04	0.1	-0.52	0.12	-0.3
51030	-0.52	0.18	-0.6	51030	1.04	-0.36	1.2	-1.55	0.54	-1.8
51033	0.17	0.82	-0.2	51033	-0.35	-1.73	0.5	0.52	2.55	-0.8
51067	-0.76	-0.48	0.2	51067	0.69	0.43	-0.2	-1.45	-0.91	0.4
51068	0.43	0.71	0.6	51068	-0.11	-0.18	-0.1	0.54	0.89	0.7
51069	0.32	-2.02	0.3	51069	-0.08	0.53	-0.1	0.40	-2.56	0.3
51121	-1.98	0.83	-0.1	51121	0.96	-0.40	0.0	-2.95	1.23	-0.1
51125	-3.49	2.21	0.7	51125	1.23	-0.78	-0.3	-4.71	2.99	1.0
51126	-3.00	0.98	0.3	51126	0.87	-0.28	-0.1	-3.86	1.26	0.3
52001	4.63	-1.61	-0.7	52001	-1.13	0.39	0.2	5.76	-2.01	-0.8
51004	2.93	-1.49	-1.0	51004	-0.98	0.50	0.3	3.91	-1.99	-1.4
51005	0.11	-1.05	0.2	51005	-0.04	0.42	-0.1	0.15	-1.48	0.3
51006	-3.34	-1.52	-1.4	51006	1.34	0.61	0.6	-4.67	-2.12	-2.0
51007	-0.25	0.34	-1.0	51007	0.08	-0.11	0.3	-0.33	0.45	-1.3
51009	-3.13	0.04	-0.7	51009	1.27	-0.02	0.3	-4.40	0.05	-1.0
51010	-1.78	-0.30	-0.3	51010	0.55	0.09	0.1	-2.33	-0.39	-0.4
51011	6.31	-6.36	-0.0	51011	-1.67	1.68	0.0	7.98	-8.03	-0.0
51013	-0.87	2.13	0.2	51013	0.21	-0.50	-0.0	-1.08	2.64	0.2
51017	5.39	-0.23	0.2	51017	-1.30	0.05	-0.1	6.68	-0.28	0.3
51019	6.65	-0.41	-0.4	51019	-1.39	0.08	0.1	8.04	-0.49	-0.5
51020	5.00	-3.12	1.5	51020	-1.01	0.63	-0.3	6.00	-3.75	1.9
51021	9.45	-1.58	-0.2	51021	-1.64	0.28	0.0	11.09	-1.86	-0.2
51022	7.15	-5.17	-0.2	51022	-1.22	0.88	0.0	8.37	-6.05	-0.3
51023	4.77	-9.17	1.9	51023	-0.77	1.48	-0.3	5.54	-10.65	2.2
51024	-0.93	-0.01	-0.2	51024	0.42	0.00	0.1	-1.35	-0.01	-0.3
51026	-1.36	0.63	0.3	51026	1.25	-0.58	-0.3	-2.61	1.21	0.6
51027	0.46	1.36	-0.8	51027	-0.62	-1.84	1.0	1.08	3.20	-1.8
51028	-0.89	-0.04	0.2	51028	0.84	0.04	-0.2	-1.73	-0.07	0.5
51029	-0.77	0.50	-0.7	51029	1.93	-1.25	1.7	-2.71	1.75	-2.4
51031	0.18	-0.05	-0.1	51031	-0.12	0.03	0.1	0.29	-0.08	-0.2
51032	1.08	0.90	-0.7	51032	-1.08	-0.90	0.7	2.15	1.80	-1.4
51095	7.69	-2.90	2.0	51095	-1.38	0.52	-0.4	9.07	-3.43	2.4

UNIT OF RESIDUALS (METERS)

Table 5-10

Transformation

NAD 27 E -TO- NWL-9D (BURSA)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-25.86	161.45	171.88	1.84	-0.24	0.46	-0.06
± 3.39	± 3.28	± 3.83	± 0.40	± 0.10	± 0.10	± 0.14

VARIANCE - COVARIANCE MATRIX

SO2= 0.67

0.115D+02	-0.242D+01	-0.353D+01	-0.251D-07	0.142D-05	0.111D-05	0.677D-06
-0.242D+01	0.107D+02	0.579D+01	0.831D-06	-0.239D-06	-0.289D-06	-0.173D-05
-0.353D+01	0.579D+01	0.147D+02	-0.604D-06	-0.387D-06	-0.444D-06	-0.240D-05
-0.251D-07	0.831D-06	-0.604D-06	0.163D-12	-0.190D-28	-0.226D-28	0.421D-28
0.142D-05	-0.239D-06	-0.387D-06	0.558D-28	0.234D-12	0.601D-13	0.740D-13
0.111D-05	-0.289D-06	-0.444D-06	0.421D-28	0.601D-13	0.215D-12	0.804D-13
0.677D-06	-0.173D-05	-0.240D-05	0.674D-28	0.740D-13	0.804D-13	0.468D-12

COEFFICIENTS OF CORRELATION

0.100D+01	-0.218D+00	-0.272D+00	-0.184D-01	0.866D+00	0.704D+00	0.292D+00
-0.218D+00	0.100D+01	0.461D+00	0.629D+00	-0.151D+00	-0.190D+00	-0.771D+00
-0.272D+00	0.461D+00	0.100D+01	-0.391D+00	-0.209D+00	-0.249D+00	-0.916D+00
-0.184D-01	0.629D+00	-0.391D+00	0.100D+01	-0.972D-16	-0.121D-15	0.153D-15
0.866D+00	-0.151D+00	-0.209D+00	-0.972D-16	0.100D+01	0.268D+00	0.223D+00
0.704D+00	-0.190D+00	-0.249D+00	-0.121D-15	0.268D+00	0.100D+01	0.253D+00
0.292D+00	-0.771D+00	-0.916D+00	0.153D-15	0.223D+00	0.253D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN GREENWICH, EAST, AND CIO. THE ROTATIONS PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXFS RESPECTIVELY.

Table 5-10 (cont'd)

RESIDUALS V
SPHERICAL

V1(NAD 27 E)				V2(NWL-9D)				V1 - V2		
10003	-1.22	-0.02	0.8	10003	0.76	0.01	-0.5	-1.98	-0.04	1.3
10018	1.56	-0.82	1.1	10018	-0.91	0.48	-0.6	2.47	-1.30	1.7
10019	0.34	1.00	1.5	10019	-0.17	-0.50	-0.7	0.51	1.50	2.2
10020	0.35	0.91	1.4	10020	-0.17	-0.44	-0.7	0.52	1.35	2.1
10021	-0.60	1.16	2.0	10021	0.30	-0.57	-1.0	-0.90	1.74	3.0
10022	-1.53	-0.29	0.3	10022	0.82	0.16	-0.1	-2.34	-0.45	0.4
10023	-1.42	-0.86	0.9	10023	0.75	0.45	-0.5	-2.16	-1.31	1.4
20016	-1.74	-1.61	2.0	20016	0.77	0.72	-0.9	-2.51	-2.33	2.9
30025	1.67	-0.13	0.7	30025	-0.58	0.05	-0.2	2.25	-0.18	0.9
30028	1.25	1.68	0.0	30028	-0.93	-1.26	-0.0	2.18	2.94	0.0
51008	0.82	-1.05	-0.7	51008	-0.20	0.25	0.2	1.02	-1.30	-0.9
51014	-2.78	-0.64	-0.6	51014	0.59	0.14	0.1	-3.37	-0.78	-0.7
51015	0.39	-0.90	-0.3	51015	-0.07	0.17	0.1	0.46	-1.07	-0.4
51025	0.45	-0.86	0.0	51025	-0.25	0.47	-0.0	0.70	-1.33	0.0
51030	0.20	0.12	-0.5	51030	-0.40	-0.24	1.0	0.60	0.35	-1.5
51033	0.69	1.28	-0.2	51033	-1.46	-2.72	0.5	2.15	4.00	-0.7
51067	0.39	-0.97	0.4	51067	-0.36	0.88	-0.4	0.74	-1.85	0.8
51068	-0.73	-0.52	0.5	51068	0.19	0.13	-0.1	-0.92	-0.66	0.6
51069	-1.04	-2.80	0.1	51069	0.28	0.74	-0.0	-1.32	-3.54	0.1
51121	-1.38	-0.20	0.1	51121	0.67	0.10	-0.1	-2.05	-0.30	0.2
51125	-3.69	1.08	0.8	51125	1.30	-0.38	-0.3	-4.99	1.46	1.1
51126	-3.77	-0.24	0.2	51126	1.09	0.07	-0.1	-4.86	-0.31	0.3
52001	2.60	-1.07	-1.0	52001	-0.64	0.26	0.3	3.24	-1.33	-1.3
51004	1.83	-1.06	-1.2	51004	-0.61	0.36	0.4	2.44	-1.42	-1.6
51005	-0.54	-0.90	0.1	51005	0.22	0.36	-0.0	-0.76	-1.26	0.1
51006	-3.87	-1.72	-1.5	51006	1.55	0.69	0.6	-5.42	-2.40	-2.1
51007	-1.31	-0.01	-1.1	51007	0.41	0.00	0.3	-1.72	-0.01	-1.4
51009	-3.40	-0.56	-0.7	51009	1.38	0.23	0.3	-4.78	-0.78	-1.0
51010	-2.69	-0.97	-0.4	51010	0.84	0.30	0.1	-3.52	-1.27	-0.5
51011	4.59	-6.17	-0.3	51011	-1.21	1.63	0.1	5.80	-7.80	-0.4
51013	-1.88	0.27	0.2	51013	0.44	-0.06	-0.0	-2.32	0.33	0.2
51017	3.28	0.74	-0.2	51017	-0.79	-0.18	0.0	4.07	0.92	-0.2
51019	4.04	0.81	-0.9	51019	-0.84	-0.17	0.2	4.88	0.98	-1.1
51020	2.31	-1.32	1.0	51020	-0.46	0.27	-0.2	2.78	-1.59	1.2
51021	6.05	-0.53	-0.8	51021	-1.05	0.09	0.1	7.10	-0.62	-0.9
51022	3.72	-3.50	-0.9	51022	-0.63	0.60	0.2	4.35	-4.10	-1.0
51023	1.15	-6.95	1.1	51023	-0.19	1.12	-0.2	1.34	-8.07	1.3
51024	0.18	-1.36	0.1	51024	-0.08	0.62	-0.0	0.26	-1.98	0.1
51026	-0.57	0.26	0.5	51026	0.52	-0.23	-0.4	-1.09	0.49	0.9
51027	1.01	1.35	-0.7	51027	-1.37	-1.83	0.9	2.38	3.18	-1.6
51028	-0.50	0.02	0.3	51028	0.47	-0.02	-0.3	-0.97	0.04	0.6
51029	-0.36	0.73	-0.6	51029	0.90	-1.83	1.6	-1.26	2.57	-2.3
51031	0.20	0.36	-0.2	51031	-0.13	-0.24	0.1	0.33	0.61	-0.3
51032	1.39	1.37	-0.7	51032	-1.39	-1.37	0.7	2.77	2.74	-1.4
51095	4.46	-1.52	1.4	51095	-0.80	0.27	-0.3	5.27	-1.80	1.7

UNIT OF RESIDUALS (METERS)

Table 5-11

Transformation

NAD 27 E -TO- NWL-9D (MOL- MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-27.44	151.61	178.54	1.84	-0.24	0.46	-0.06
± 0.36	± 0.37	± 0.37	± 0.40	± 0.10	± 0.10	± 0.14

VARIANCE - COVARIANCE MATRIX

SO2= 0.67

0.130D+00	0.772D-03	-0.100D-02	0.600D-07	0.179D-08	-0.191D-07	-0.543D-08
0.772D-03	0.137D+00	-0.537D-02	0.532D-08	-0.791D-07	-0.143D-07	0.184D-07
-0.100D-02	-0.537D-02	0.136D+00	0.159D-07	0.198D-07	0.769D-07	0.143D-07
0.600D-07	0.532D-08	0.159D-07	0.163D-12	0.105D-29	-0.128D-29	-0.181D-29
0.179D-08	-0.791D-07	0.198D-07	-0.105D-29	0.234D-12	0.601D-13	0.740D-13
-0.191D-07	-0.143D-07	0.769D-07	0.122D-29	0.601D-13	0.215D-12	0.804D-13
-0.543D-08	0.184D-07	0.143D-07	0.266D-29	0.740D-13	0.804D-13	0.468D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.577D-02	-0.755D-02	0.412D+00	0.103D-01	-0.114D+00	-0.220D-01
0.577D-02	0.100D+01	-0.393D-01	0.356D-01	-0.441D+00	-0.832D-01	0.726D-01
-0.755D-02	-0.393D-01	0.100D+01	0.107D+00	0.111D+00	0.450D+00	0.569D-01
0.412D+00	0.356D-01	0.107D+00	0.100D+01	0.540D-17	-0.686D-17	-0.656D-17
0.103D-01	-0.441D+00	0.111D+00	0.540D-17	0.100D+01	0.268D+00	0.223D+00
-0.114D+00	-0.832D-01	0.450D+00	-0.686D-17	0.268D+00	0.100D+01	0.253D+00
-0.220D-01	0.726D-01	0.569D-01	-0.656D-17	0.223D+00	0.253D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-12

Transformation

NAD 27 W -TO- NWL-9D (3 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION PARAMETERS
(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS
-0.28480391D+02	0.155225890D+03	0.17996662D+03
± 0.67	± 0.67	± 0.67

VARIANCE - COVARIANCE MATRIX

MO2= 1.89

0.45494455D+00	0.0	0.0
0.0	0.45494455D+00	0.0
0.0	0.0	0.45494455D+00

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.0	0.0
0.0	0.10000000D+01	0.0
0.0	0.0	0.10000000D+01

Table 5-12 (cont'd)

RESIDUALS V
SPHERICAL

V1(NAD 27 W)				V2(NWL-9D)				V1 - V2		
10000	0.91	1.84	0.9	10000	-1.06	-2.14	-1.1	1.97	3.98	2.0
10006	-1.77	1.81	1.5	10006	1.00	-1.02	-0.9	-2.76	2.93	2.4
10031	-2.00	-4.67	2.8	10031	0.57	1.32	-0.8	-2.57	-5.99	3.5
10055	5.05	-7.93	0.9	10055	-1.06	1.67	-0.2	6.11	-9.60	1.1
20003	-1.13	-5.53	0.7	20003	0.30	1.44	-0.2	-1.43	-6.96	0.9
30029	6.29	-2.78	0.1	30029	-1.51	0.67	-0.0	7.80	-3.45	0.1
30098	3.78	-6.12	-0.3	30098	-0.83	1.35	0.1	4.62	-7.47	-0.4
30099	7.52	4.09	0.2	30099	-3.15	-1.71	-0.1	10.67	5.80	0.3
51039	-4.44	-0.05	0.9	51039	2.58	0.03	-0.5	-7.01	-0.08	1.4
51041	-0.25	0.85	0.3	51041	0.52	-1.78	-0.6	-0.77	2.63	1.0
51043	2.90	3.98	0.4	51043	-1.59	-2.18	-0.2	4.50	6.16	0.6
51044	0.99	2.19	-0.1	51044	-0.72	-1.59	0.1	1.71	3.78	-0.2
51048	-2.11	-0.33	0.3	51048	1.48	0.23	-0.2	-3.58	-0.56	0.5
51052	-3.55	0.03	-1.8	51052	1.31	-0.01	0.7	-4.86	0.04	-2.4
51066	4.64	-4.68	-0.6	51066	-1.06	1.07	0.1	5.70	-5.75	-0.7
51074	5.74	-2.45	-0.5	51074	-1.30	0.55	0.1	7.03	-3.00	-0.6
51089	3.25	-6.04	-0.8	51089	-0.72	1.34	0.2	3.97	-7.39	-1.0
51103	-4.72	-0.35	0.2	51103	2.43	0.18	-0.1	-7.15	-0.53	0.3
51123	-4.88	0.60	1.9	51123	2.51	-0.31	-1.0	-7.39	0.91	2.9
51124	-4.20	0.63	1.3	51124	2.64	-0.40	-0.8	-6.84	1.03	2.2
52063	3.33	-4.95	0.2	52063	-0.78	1.15	-0.0	4.11	-6.10	0.3
10008	0.79	3.61	-0.2	10008	-0.46	-2.11	0.1	1.25	5.72	-0.3
10056	-0.94	-6.39	-1.2	10056	0.21	1.44	0.3	-1.16	-7.83	-1.4
10071	-1.72	-0.73	1.4	10071	1.00	0.42	-0.8	-2.72	-1.15	2.1
20176	-3.10	-0.80	-1.2	20176	1.03	0.26	0.4	-4.13	-1.06	-1.5
20177	-4.58	-1.71	-1.2	20177	1.78	0.66	0.5	-6.36	-2.38	-1.7
20208	-1.42	-3.30	1.2	20208	0.49	1.14	-0.4	-1.91	-4.45	1.7
30030	1.29	0.57	1.3	30030	-0.73	-0.32	-0.7	2.02	0.89	2.0
51056	1.91	0.58	0.6	51056	-0.89	-0.27	-0.3	2.80	0.84	0.8
51057	2.57	-0.63	0.1	51057	-0.90	0.22	-0.0	3.48	-0.86	0.1
51058	3.29	-3.50	-1.2	51058	-0.88	0.93	0.3	4.17	-4.43	-1.5

UNIT OF RESIDUALS (METERS)

Table 5-13

Transformation

NAD 27 W -TO- NWL-9D (4 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION AND 1 SCALE PARAMETERS

(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)
-0.18682805D+02	0.18308447D+03	0.15673465D+03	0.59502010D+01
± 0.92	± 2.36	± 1.98	± 0.50

VARIANCE - COVARIANCE MATRIX

M02= 0.73

0.84237019D+00	0.18955240D+01	-0.15807250D+01	0.40485736D-06
0.18955240D+01	0.55654894D+01	-0.44946529D+01	0.11511764D-05
-0.15807250D+01	-0.44946529D+01	0.39239369D+01	-0.95999486D-06
0.40485736D-06	0.11511764D-05	-0.95999486D-06	0.24587514D-12

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.87543946D+00	-0.86944843D+00	0.88959685D+00
0.87543946D+00	0.10000000D+01	-0.96179712D+00	0.98408562D+00
-0.86944843D+00	-0.96179712D+00	0.10000000D+01	-0.97735106D+00
0.88959685D+00	0.98408562D+00	-0.97735106D+00	0.10000000D+01

Table 5-13 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD 27 W)				V2(NWL-9D)			V1 - V2		
10000	0.10	0.77	0.7	10000	-0.12	-0.89	-0.9	0.22	1.66	1.6
10006	-1.91	-1.74	1.0	10006	1.07	0.98	-0.6	-2.98	-2.72	1.6
10031	-0.38	-1.66	2.3	10031	0.11	0.47	-0.7	-0.49	-2.13	2.9
10055	5.91	-2.39	0.1	10055	-1.25	0.50	-0.0	7.16	-2.90	0.1
20003	1.08	-2.14	0.2	20003	-0.28	0.56	-0.1	1.36	-2.70	0.2
30029	1.92	1.34	-0.8	30029	-0.46	-0.32	0.2	2.38	1.66	-1.0
30098	3.43	-0.77	-1.0	30098	-0.76	0.17	0.2	4.19	-0.94	-1.3
30099	3.20	3.82	-0.4	30099	-1.34	-1.60	0.1	4.54	5.41	-0.5
51039	-1.26	-2.47	0.3	51039	0.73	1.44	-0.2	-1.99	-3.91	0.5
51041	-0.88	-0.44	0.1	51041	1.84	0.93	-0.2	-2.72	-1.37	0.4
51043	0.20	3.63	0.0	51043	-0.11	-1.99	-0.0	0.31	5.63	0.1
51044	-0.23	1.72	-0.3	51044	0.17	-1.24	0.3	-0.40	2.96	-0.6
51048	-0.77	-1.21	0.0	51048	0.54	0.85	-0.0	-1.31	-2.06	0.0
51052	-0.84	0.83	-2.1	51052	0.31	-0.31	0.8	-1.15	1.14	-2.9
51066	1.80	0.28	-1.4	51066	-0.41	-0.06	0.3	2.21	0.35	-1.7
51074	4.12	2.74	-1.2	51074	-0.93	-0.62	0.3	5.05	3.36	-1.5
51089	3.72	-0.88	-1.5	51089	-0.83	0.19	0.3	4.55	-1.07	-1.9
51103	-1.98	-1.33	-0.2	51103	1.02	0.69	0.1	-3.00	-2.02	-0.3
51123	-1.51	-1.22	1.4	51123	0.78	0.63	-0.7	-2.29	-1.85	2.1
51124	-1.13	-2.30	0.7	51124	0.71	1.45	-0.5	-1.84	-3.74	1.2
52063	4.39	-0.30	-0.4	52063	-1.02	0.07	0.1	5.41	-0.38	-0.5
10008	-2.84	-0.26	-1.0	10008	1.66	0.15	0.6	-4.50	-0.41	-1.6
10056	2.00	-2.14	-1.9	10056	-0.45	0.48	0.4	2.45	-2.62	-2.3
10071	-0.53	-0.98	1.1	10071	0.31	0.57	-0.6	-0.84	-1.55	1.7
20176	-0.17	0.55	-1.6	20176	0.06	-0.18	0.5	-0.22	0.73	-2.1
20177	-1.28	-1.65	-1.7	20177	0.50	0.64	0.7	-1.78	-2.29	-2.3
20208	0.17	-1.52	0.9	20208	-0.06	0.53	-0.3	0.23	-2.04	1.2
30030	1.04	0.81	1.1	30030	-0.59	-0.46	-0.6	1.63	1.26	1.6
51056	0.92	1.49	0.3	51056	-0.42	-0.69	-0.1	1.34	2.18	0.4
51057	1.67	1.59	-0.3	51057	-0.58	-0.56	0.1	2.26	2.14	-0.4
51058	2.75	0.40	-1.7	51058	-0.73	-0.11	0.5	3.49	0.50	-2.2

UNIT OF RESIDUALS (METERS)

Table 5-14

Transformation

NAD 27 W -TO- NWL-9D (BURSA)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-12.92	180.17	155.67	5.95	0.60	-0.41	-0.10
± 2.92	± 2.59	± 2.83	± 0.34	± 0.09	± 0.08	± 0.10

VARIANCE - COVARIANCE MATRIX

SO2= 0.34

0.852D+01	-0.328D+00	0.124D+01	0.189D-06	0.998D-06	0.885D-06	-0.111D-06
-0.328D+00	0.672D+01	0.234D+01	0.536D-06	-0.311D-06	0.624D-07	-0.926D-06
0.124D+01	0.234D+01	0.798D+01	-0.447D-06	0.482D-07	0.448D-06	-0.116D-05
0.189D-06	0.536D-06	-0.447D-06	0.115D-12	0.814D-28	0.240D-27	-0.274D-27
0.998D-06	-0.311D-06	0.482D-07	0.686D-28	0.173D-12	0.480D-13	0.659D-14
0.885D-06	0.624D-07	0.448D-06	0.107D-27	0.480D-13	0.169D-12	-0.362D-13
-0.111D-06	-0.926D-06	-0.116D-05	-0.211D-27	0.659D-14	-0.362D-13	0.234D-12

COEFFICIENTS OF CORRELATION

0.100D+01	-0.433D-01	0.150D+00	0.191D+00	0.822D+00	0.737D+00	-0.783D-01
-0.433D-01	0.100D+01	0.320D+00	0.611D+00	-0.288D+00	0.585D-01	-0.738D+00
0.150D+00	0.320D+00	0.100D+01	-0.468D+00	0.410D-01	0.386D+00	-0.846D+00
0.191D+00	0.611D+00	-0.468D+00	0.100D+01	0.578D-15	0.172D-14	-0.167D-14
0.822D+00	-0.288D+00	0.410D-01	0.578D-15	0.100D+01	0.281D+00	0.327D-01
0.737D+00	0.585D-01	0.386D+00	0.172D-14	0.281D+00	0.100D+01	-0.182D+00
-0.783D-01	-0.738D+00	-0.846D+00	-0.167D-14	0.327D-01	-0.182D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN GREENWICH, EAST, AND CIO. THE ROTATIONS PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-14 (cont'd)

RESIDUALS V
SPHERICAL

	V1(NAD 27 W)				V2(NWL-9D)			V1 - V2		
10000	0.72	0.33	0.6	10000	-0.84	-0.39	-0.7	1.56	0.72	1.3
10006	0.13	-1.74	0.5	10006	-0.08	0.98	-0.3	0.21	-2.72	0.7
10031	-2.08	-0.67	2.8	10031	0.59	0.19	-0.8	-2.67	-0.86	3.6
10055	2.77	-1.77	1.0	10055	-0.58	0.37	-0.2	3.35	-2.14	1.3
20003	-0.82	-0.81	0.7	20003	0.21	0.21	-0.2	-1.03	-1.02	0.9
30029	-0.55	-1.12	-0.0	30029	0.13	0.27	0.0	-0.68	-1.39	-0.0
30098	0.36	-0.84	-0.1	30098	-0.08	0.19	0.0	0.44	-1.03	-0.1
30099	3.34	1.32	-0.2	30099	-1.40	-0.55	0.1	4.74	1.87	-0.3
51039	0.06	-0.61	-0.2	51039	-0.04	0.36	0.1	0.10	-0.97	-0.4
51041	-0.13	-0.78	-0.1	51041	0.27	1.64	0.1	-0.40	-2.42	-0.2
51043	0.39	2.09	0.1	51043	-0.21	-1.15	-0.0	0.60	3.24	0.1
51044	0.04	1.04	-0.4	51044	-0.03	-0.75	0.3	0.07	1.79	-0.7
51048	-0.28	-0.42	-0.2	51048	0.20	0.29	0.1	-0.48	-0.71	-0.3
51052	-1.30	2.40	-2.1	51052	0.48	-0.89	0.8	-1.78	3.29	-2.9
51066	-1.12	-1.24	-0.5	51066	0.26	0.29	0.1	-1.38	-1.53	-0.7
51074	1.10	1.93	-0.3	51074	-0.25	-0.44	0.1	1.35	2.37	-0.4
51089	0.77	-0.49	-0.6	51089	-0.17	0.11	0.1	0.95	-0.59	-0.8
51103	-1.45	0.25	-0.4	51103	0.75	-0.13	0.2	-2.20	0.38	-0.7
51123	-0.53	0.73	0.9	51123	0.27	-0.38	-0.5	-0.80	1.11	1.4
51124	0.47	-0.49	0.1	51124	-0.30	0.31	-0.1	0.77	-0.80	0.1
52063	1.75	0.40	0.4	52063	-0.41	-0.09	-0.1	2.16	0.49	0.4
10008	-0.53	-2.34	-1.4	10008	0.31	1.37	0.8	-0.85	-3.71	-2.2
10056	-0.36	-0.39	-1.2	10056	0.08	0.09	0.3	-0.44	-0.48	-1.5
10071	-0.40	-0.27	1.0	10071	0.23	0.15	-0.6	-0.63	-0.42	1.6
20176	-0.93	2.25	-1.5	20176	0.31	-0.74	0.5	-1.23	2.99	-1.9
20177	-1.33	0.24	-1.8	20177	0.52	-0.09	0.7	-1.85	0.34	-2.5
20208.	-0.84	-0.57	1.1	20208	0.29	0.20	-0.4	-1.13	-0.76	1.5
30030	0.90	0.70	1.1	30030	-0.50	-0.40	-0.6	1.40	1.10	1.7
51056	0.38	0.96	0.5	51056	-0.18	-0.44	-0.2	0.55	1.40	0.7
51057	0.38	1.13	0.1	51057	-0.13	-0.39	-0.0	0.51	1.52	0.2
51058	0.51	0.18	-1.1	51058	-0.14	-0.05	0.3	0.64	0.22	-1.3

UNIT OF RESIDUALS (METERS)

Table 5-15

Transformation

NAD 27 W -TO- NWL-9D (MOL- MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-28.62	155.47	180.61	5.95	0.60	-0.41	-0.10
± 0.29	± 0.29	± 0.29	± 0.34	± 0.09	± 0.09	± 0.10

VARIANCE - COVARIANCE MATRIX

SO2= 0.34

0.835D-01	0.965D-04	0.171D-03	-0.697D-08	-0.915D-09	-0.138D-07	0.327D-08
0.965D-04	0.844D-01	-0.190D-03	0.215D-08	0.111D-07	-0.217D-09	0.207D-07
0.171D-03	-0.190D-03	0.834D-01	0.993D-08	-0.305D-08	-0.961D-09	-0.220D-08
-0.697D-08	0.215D-08	0.993D-08	0.115D-12	-0.887D-30	-0.756D-30	0.159D-29
-0.915D-09	0.111D-07	-0.305D-08	0.144D-29	0.173D-12	0.480D-13	0.659D-14
-0.138D-07	-0.217D-09	-0.961D-08	0.108D-29	0.480D-13	0.169D-12	-0.362D-13
0.327D-08	0.207D-07	-0.220D-08	0.954D-30	0.659D-14	-0.362D-13	0.234D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.115D-02	0.206D-02	-0.713D-01	-0.761D-02	-0.116D+00	0.233D-01
0.115D-02	0.100D+01	-0.226D-02	0.219D-01	0.919D-01	-0.182D-02	0.147D+00
0.206D-02	-0.226D-02	0.100D+01	0.102D+00	-0.254D-01	-0.810D-01	-0.157D-01
-0.713D-01	0.219D-01	0.102D+00	0.100D+01	-0.630D-17	-0.543D-17	0.970D-17
-0.761D-02	0.919D-01	-0.254D-01	-0.630D-17	0.100D+01	0.281D+00	0.327D-01
-0.116D+00	-0.182D-02	-0.810D-01	-0.543D-17	0.281D+00	0.100D+01	-0.182D+00
0.233D-01	0.147D+00	-0.157D-01	0.970D-17	0.327D-01	-0.182D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN GREENWICH, EAST, AND CIO. THE ROTATIONS PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-16

Transformation

TRAVERS -TO- NWL-9D (3 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION PARAMETERS
(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS
-0.26838255D+02	0.15142910D+03	0.17779137D+03
± 0.26	± 0.26	± 0.26

VARIANCE - COVARIANCE MATRIX

MO2= 1.48

0.672187830-01	0.0	0.0
0.0	0.672187830-01	0.0
0.0	0.0	0.672187830-01

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.0	0.0
0.0	0.10000000D+01	0.0
0.0	0.0	0.10000000D+01

Table 5-16 (cont'd)

RESIDUALS V
SPHERICAL

	V1 (TRAVERS)				V2 (NWL-9D)			V1 - V2		
10000	0.34	-0.91	0.5	10000	-0.34	0.91	-0.5	0.68	-1.81	1.0
10003	-0.50	0.72	0.7	10003	0.50	-0.72	-0.7	-1.00	1.44	1.4
10006	0.57	0.32	0.5	10006	-0.57	-0.32	-0.5	1.14	0.65	1.0
10018	0.37	-0.15	0.9	10018	-0.37	0.15	-0.9	0.74	-0.30	1.7
10019	-0.40	0.32	1.2	10019	0.40	-0.32	-1.2	-0.79	0.64	2.5
10020	-0.45	0.04	1.2	10020	0.45	-0.04	-1.2	-0.89	0.07	2.4
10021	-0.41	0.86	1.6	10021	0.41	-0.86	-1.6	-0.83	1.72	3.3
10022	-0.49	0.81	0.3	10022	0.49	-0.81	-0.3	-0.98	1.61	0.5
10023	-0.76	0.35	0.7	10023	0.76	-0.35	-0.7	-1.51	0.69	1.5
10031	1.05	-1.35	1.3	10031	-1.05	1.35	-1.3	2.10	-2.69	2.6
10055	0.82	-1.23	0.3	10055	-0.82	1.23	-0.3	1.63	-2.46	0.5
20003	1.19	-1.42	0.0	20003	-1.19	1.42	-0.0	2.39	-2.85	0.0
20016	-0.97	0.24	1.5	20016	0.97	-0.24	-1.5	-1.95	0.49	3.0
30025	-0.41	0.08	0.6	30025	0.41	-0.08	-0.6	-0.82	0.16	1.3
30028	0.00	0.55	0.1	30028	-0.00	-0.55	-0.1	0.00	1.10	0.3
30029	1.59	-2.63	0.0	30029	-1.59	2.63	-0.0	3.18	-5.26	0.0
30098	0.84	-1.96	-0.4	30098	-0.84	1.96	0.4	1.68	-3.93	-0.8
30099	-0.53	-1.85	-0.1	30099	0.53	1.85	0.1	-1.07	-3.70	-0.1
51008	-0.90	-0.20	-0.2	51008	0.90	0.20	0.2	-1.80	-0.40	-0.5
51014	-1.21	0.24	-0.2	51014	1.21	-0.24	0.2	-2.41	0.48	-0.4
51015	-1.11	0.20	0.0	51015	1.11	-0.20	-0.0	-2.22	0.40	0.0
51025	-0.45	-0.03	0.0	51025	0.45	0.03	-0.0	-0.90	-0.06	0.1
51030	0.69	0.36	-0.7	51030	-0.69	-0.36	0.7	1.38	0.71	-1.4
51033	0.71	0.35	-0.2	51033	-0.71	-0.35	0.2	1.42	0.70	-0.4
51039	-0.46	0.33	-0.2	51039	0.46	-0.33	0.2	-0.92	0.66	-0.4
51041	-0.53	-0.55	-0.1	51041	0.53	0.55	0.1	-1.06	-1.11	-0.1
51043	-0.88	-1.06	-0.0	51043	0.88	1.06	0.0	-1.76	-2.11	-0.0
51044	-0.88	-0.41	-0.5	51044	0.88	0.41	0.5	-1.75	-0.82	-1.1
51048	1.18	1.02	-0.4	51048	-1.18	-1.02	0.4	2.36	2.04	-0.9
51052	1.06	0.64	-1.9	51052	-1.06	-0.64	1.9	2.12	1.28	-3.7
51066	1.08	-3.01	-0.5	51066	-1.08	3.01	0.5	2.17	-6.03	-0.9
51067	-0.05	-0.27	0.4	51067	0.05	0.27	-0.4	-0.10	-0.53	0.8
51068	-1.10	0.64	0.5	51068	1.10	-0.64	-0.5	-2.20	1.27	0.9
51069	-1.11	0.62	0.2	51069	1.11	-0.62	-0.2	-2.22	1.24	0.5
51074	1.07	-1.17	-0.4	51074	-1.07	1.17	0.4	2.15	-2.33	-0.9
51089	0.64	-1.13	-0.8	51089	-0.64	1.13	0.8	1.28	-2.26	-1.5
51103	0.92	0.78	-0.6	51103	-0.92	-0.78	0.6	1.84	1.57	-1.2
51121	-1.09	0.08	0.1	51121	1.09	-0.08	-0.1	-2.18	0.16	0.2
51123	0.14	0.78	0.6	51123	-0.14	-0.78	-0.6	0.29	1.57	1.2
51124	-0.69	0.86	0.2	51124	0.69	-0.86	-0.2	-1.38	1.72	0.3
51125	-1.53	0.68	0.6	51125	1.53	-0.68	-0.6	-3.07	1.37	1.2
51126	-1.34	0.43	0.3	51126	1.34	-0.43	-0.3	-2.68	0.87	0.5
52001	-0.97	-0.09	-0.4	52001	0.97	0.09	0.4	-1.94	-0.18	-0.7
52063	1.28	-0.06	-0.2	52063	-1.28	0.06	0.2	2.57	-0.11	-0.4

UNIT OF RESIDUALS (METERS)

Table 5-17

Transformation

TRAVERS -TO- NWL-9D (BURSA)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-23.58	155.71	173.39	1.03	0.15	-0.05	0.02
± 1.31	± 1.59	± 2.05	± 0.16	± 0.04	± 0.04	± 0.08

VARIANCE - COVARIANCE MATRIX

SO2= 0.99

0.172D+01	0.928D+00	0.138D+01	0.178D-07	0.191D-06	0.194D-06	-0.264D-06
0.928D+00	0.252D+01	0.223D+01	0.119D-06	0.645D-07	0.140D-06	-0.521D-06
0.138D+01	0.223D+01	0.421D+01	-0.896D-07	0.124D-06	0.224D-06	-0.743D-06
0.178D-07	0.119D-06	-0.896D-07	0.241D-13	0.441D-29	0.313D-29	0.133D-28
0.191D-06	0.645D-07	0.124D-06	0.127D-28	0.314D-13	0.987D-14	-0.236D-13
0.194D-06	0.140D-06	0.224D-06	0.940D-29	0.987D-14	0.390D-13	-0.396D-13
-0.264D-06	-0.521D-06	-0.743D-06	0.470D-29	-0.236D-13	-0.396D-13	0.145D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.445D+00	0.511D+00	0.874D-01	0.823D+00	0.748D+00	-0.528D+00
0.445D+00	0.100D+01	0.685D+00	0.482D+00	0.229D+00	0.447D+00	-0.863D+00
0.511D+00	0.685D+00	0.100D+01	-0.281D+00	0.340D+00	0.553D+00	-0.952D+00
0.874D-01	0.482D+00	-0.281D+00	0.100D+01	0.160D-15	0.102D-15	0.226D-15
0.823D+00	0.229D+00	0.340D+00	0.160D-15	0.100D+01	0.282D+00	-0.350D+00
0.748D+00	0.447D+00	0.553D+00	0.102D-15	0.282D+00	0.100D+01	-0.527D+00
-0.528D+00	-0.863D+00	-0.952D+00	0.226D-15	-0.350D+00	-0.527D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-17 (cont'd)

RESIDUALS V
SPHERICAL

	V1(TRAVERS)				V2(NWL-9D)			V1 - V2		
10000	-0.07	-0.73	0.6	10000	0.07	0.73	-0.6	-0.15	-1.46	1.1
10003	-0.10	0.55	0.4	10003	0.10	-0.55	-0.4	-0.20	1.10	0.8
10006	0.43	0.27	0.4	10006	-0.43	-0.27	-0.4	0.87	0.55	0.9
10018	0.73	0.07	0.7	10018	-0.73	-0.07	-0.7	1.47	0.15	1.4
10019	-0.18	-0.29	0.9	10019	0.18	0.29	-0.9	-0.37	-0.58	1.9
10020	-0.14	-0.52	0.8	10020	0.14	0.52	-0.8	-0.28	-1.05	1.7
10021	-0.07	0.36	1.3	10021	0.07	-0.36	-1.3	-0.14	0.71	2.6
10022	-0.08	0.48	-0.0	10022	0.08	-0.48	0.0	-0.15	0.96	-0.1
10023	-0.31	0.10	0.4	10023	0.31	-0.10	-0.4	-0.62	0.20	0.9
10031	0.73	-0.36	1.5	10031	-0.73	0.36	-1.5	1.45	-0.73	2.9
10055	0.29	-0.02	0.5	10055	-0.29	0.02	-0.5	0.57	-0.04	0.9
20003	0.92	-0.37	0.2	20003	-0.92	0.37	-0.2	1.84	-0.74	0.3
20016	-0.42	0.10	1.2	20016	0.42	-0.10	-1.2	-0.84	0.19	2.3
30025	-0.04	-0.67	0.2	30025	0.04	0.67	-0.2	-0.09	-1.35	0.4
30028	0.06	0.05	-0.1	30028	-0.06	-0.05	0.1	0.13	0.10	-0.2
30029	0.52	-1.92	0.1	30029	-0.52	1.92	-0.1	1.04	-3.85	0.3
30098	0.18	-0.84	-0.2	30098	-0.18	0.84	0.2	0.36	-1.69	-0.4
30099	-1.43	-1.73	0.0	30099	1.43	1.73	-0.0	-2.86	-3.45	0.0
51008	-0.11	-0.85	-0.8	51008	0.11	0.85	0.8	-0.22	-1.71	-1.6
51014	-0.21	-0.13	-0.8	51014	0.21	0.13	0.8	-0.43	-0.26	-1.7
51015	-0.02	-0.12	-0.7	51015	0.02	0.12	0.7	-0.03	-0.23	-1.3
51025	0.01	-0.01	-0.2	51025	-0.01	0.01	0.2	0.02	-0.02	-0.4
51030	0.76	0.40	-0.8	51030	-0.76	-0.40	0.8	1.52	0.80	-1.6
51033	0.52	0.08	-0.3	51033	-0.52	-0.08	0.3	1.04	0.16	-0.7
51039	-0.22	0.72	-0.3	51039	0.22	-0.72	0.3	-0.43	1.44	-0.6
51041	-0.89	-0.55	-0.1	51041	0.89	0.55	0.1	-1.77	-1.10	-0.1
51043	-1.59	-0.84	0.1	51043	1.59	0.84	-0.1	-3.18	-1.67	0.1
51044	-1.39	-0.11	-0.5	51044	1.39	0.11	0.5	-2.78	-0.21	-0.9
51048	1.09	1.49	-0.4	51048	-1.09	-1.49	0.4	2.17	2.98	-0.8
51052	0.99	1.43	-1.8	51052	-0.99	-1.43	1.8	1.98	2.87	-3.5
51066	0.15	-2.10	-0.3	51066	-0.15	2.10	0.3	0.30	-4.20	-0.6
51067	0.18	-0.12	0.3	51067	-0.18	0.12	-0.3	0.35	-0.24	0.6
51068	-0.27	0.21	-0.1	51068	0.27	-0.21	0.1	-0.55	0.43	-0.2
51069	-0.34	0.09	-0.3	51069	0.34	-0.09	0.3	-0.68	0.17	-0.6
51074	0.27	-0.14	-0.2	51074	-0.27	0.14	0.2	0.54	-0.29	-0.5
51089	0.08	0.02	-0.6	51089	-0.08	-0.02	0.6	0.16	0.04	-1.2
51103	0.99	1.35	-0.6	51103	-0.99	-1.35	0.6	1.98	2.70	-1.2
51121	-0.58	0.05	-0.2	51121	0.58	-0.05	0.2	-1.16	0.10	-0.3
51123	0.35	1.27	0.5	51123	-0.35	-1.27	-0.5	0.70	2.55	1.1
51124	-0.41	1.16	0.1	51124	0.41	-1.16	-0.1	-0.82	2.31	0.1
51125	-0.87	0.45	0.2	51125	0.87	-0.45	-0.2	-1.74	0.90	0.4
51126	-0.57	0.09	-0.2	51126	0.57	-0.09	0.2	-1.14	0.17	-0.4
52001	-0.37	-0.96	-0.9	52001	0.37	0.96	0.9	-0.74	-1.92	-1.9
52063	0.81	1.07	-0.0	52063	-0.81	-1.07	0.0	1.63	2.14	-0.0

UNIT OF RESIDUALS (METERS)

Table 5-18

Transformation

TRAVERS -TO- NWL-9D (MOL- MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-26.73	151.49	178.09	1.03	0.15	-0.05	0.02
± 0.22	± 0.24	± 0.22	± 0.16	± 0.04	± 0.04	± 0.08

VARIANCE - COVARIANCE MATRIX

SO2= 0.99

0.482D-01	0.307D-02	-0.421D-03	0.118D-09	-0.166D-08	-0.109D-07	0.106D-07
0.307D-02	0.574D-01	-0.139D-02	0.915D-09	-0.697D-08	-0.115D-07	0.420D-07
-0.421D-03	-0.139D-02	0.474D-01	0.696D-08	0.945D-09	0.170D-08	-0.570D-08
0.118D-09	0.915D-09	0.696D-08	0.241D-13	0.0	0.0	0.979D-31
-0.166D-08	-0.697D-08	0.945D-09	-0.612D-32	0.314D-13	0.987D-14	-0.236D-13
-0.109D-07	-0.115D-07	0.170D-08	-0.612D-32	0.987D-14	0.390D-13	-0.396D-13
0.106D-07	0.420D-07	-0.570D-08	0.147D-30	-0.236D-13	-0.396D-13	0.145D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.583D-01	-0.881D-02	0.346D-02	-0.426D-01	-0.251D+00	0.126D+00
0.583D-01	0.100D+01	-0.266D-01	0.246D-01	-0.164D+00	-0.243D+00	0.461D+00
-0.881D-02	-0.266D-01	0.100D+01	0.206D+00	0.245D-01	0.395D-01	-0.688D-01
0.346D-02	0.246D-01	0.206D+00	0.100D+01	0.0	0.0	0.166D-17
-0.426D-01	-0.164D+00	0.245D-01	0.0	0.100D+01	0.282D+00	-0.350D+00
-0.251D+00	-0.243D+00	0.395D-01	0.0	0.282D+00	0.100D+01	-0.527D+00
0.126D+00	0.461D+00	-0.688D-01	0.166D-17	-0.350D+00	-0.527D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN GREENWICH, EAST, AND CIO. THE ROTATIONS PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-19

Transformation

TRAVERS -TO- NWL-9D (VEIS MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	ALPHA SECONDS	KSI SECONDS	ETA SECONDS
-26.73	151.49	178.09	1.03	0.13	0.02	-0.08
± 0.22	± 0.24	± 0.22	± 0.16	± 0.03	± 0.08	± 0.04

VARIANCE - COVARIANCE MATRIX

SO2= 0.99

0.482D-01	0.307D-02	-0.421D-03	0.118D-09	0.609D-08	0.121D-07	0.711D-08
0.307D-02	0.574D-01	-0.139D-02	0.915D-09	-0.421D-09	0.432D-07	0.865D-08
-0.421D-03	-0.139D-02	0.474D-01	0.696D-08	-0.478D-10	-0.589D-08	-0.126D-08
0.118D-09	0.915D-09	0.696D-08	0.241D-13	0.463D-30	-0.490D-30	-0.634D-30
0.609D-08	-0.421D-09	-0.478D-10	0.464D-30	0.242D-13	0.165D-14	0.433D-15
0.121D-07	0.432D-07	-0.589D-08	-0.392D-30	0.165D-14	0.154D-12	0.333D-13
0.711D-08	0.865D-08	-0.126D-08	-0.569D-30	0.433D-15	0.333D-13	0.368D-13

COEFFICIENTS OF CORRELATION

0.100D+01	0.583D-01	-0.881D-02	0.346D-02	0.178D+00	0.140D+00	0.169D+00
0.583D-01	0.100D+01	-0.266D-01	0.246D-01	-0.113D-01	0.460D+00	0.188D+00
-0.881D-02	-0.266D-01	0.100D+01	0.206D+00	-0.141D-02	-0.689D-01	-0.301D-01
0.346D-02	0.246D-01	0.206D+00	0.100D+01	0.192D-16	-0.804D-17	-0.213D-16
0.178D+00	-0.113D-01	-0.141D-02	0.192D-16	0.100D+01	0.269D-01	0.145D-01
0.140D+00	0.460D+00	-0.689D-01	-0.804D-17	0.269D-01	0.100D+01	0.442D+00
0.169D+00	0.188D+00	-0.301D-01	-0.213D-16	0.145D-01	0.442D+00	0.100D+01

Table 5-20

Transformation

AUSNAT -TO- NWL-9D (3 PARAMETER)

****

SOLUTION FOR 3 TRANSLATION PARAMETERS
(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS
-0.12307143D+03	-0.30657143D+02	0.14120000D+03
± 1.47	± 1.47	± 1.47

VARIANCE - COVARIANCE MATRIX

MO2= 1.34

0.21721542D+01	0.0	0.0
0.0	0.21721542D+01	0.0
0.0	0.0	0.21721542D+01

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.0	0.0
0.0	0.10000000D+01	0.0
0.0	0.0	0.10000000D+01

Table 5-20 (cont'd)

RESIDUALS V
SPHERICAL

	V1(AUSNAT)				V2(NWL-9D)			V1 - V2		
21120	1.89	-0.24	-0.2	21120	-0.47	0.06	0.1	2.36	-0.30	-0.3
27070	-0.53	-6.61	1.0	27070	0.13	1.63	-0.2	-0.66	-8.24	1.2
27090	-7.70	2.77	1.6	27090	1.90	-0.68	-0.4	-9.60	3.45	1.9
27250	2.83	1.91	-1.4	27250	-0.70	-0.47	0.3	3.53	2.38	-1.7
27430	1.04	-0.99	-0.1	27430	-0.26	0.25	0.0	1.29	-1.24	-0.2
27490	1.60	-3.01	-1.1	27490	-0.40	0.74	0.3	2.00	-3.76	-1.3
28050	1.00	5.27	0.9	28050	-0.25	-1.30	-0.2	1.25	6.57	1.1

UNIT OF RESIDUALS (METERS)

Table 5-21

Transformation

AUSNAT -TO- NWL-9D (BURSA)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-115.46	-28.69	138.09	0.36	0.07	-0.52	0.35
± 7.15	± 6.95	± 8.27	± 0.91	± 0.24	± 0.25	± 0.25

VARIANCE - COVARIANCE MATRIX

SO2= 0.98

0.511D+02	0.646D+01	-0.166D+02	0.337D-05	-0.552D-05	-0.531D-05	-0.130D-05
0.646D+01	0.483D+02	0.956D+01	-0.301D-05	-0.553D-05	0.494D-06	0.448D-05
-0.166D+02	0.956D+01	0.684D+02	0.242D-05	0.804D-06	0.801D-05	0.738D-05
0.337D-05	-0.301D-05	0.242D-05	0.823D-12	-0.119D-26	0.266D-26	0.159D-26
-0.552D-05	-0.553D-05	0.804D-06	-0.152D-26	0.130D-11	0.256D-12	-0.668D-13
-0.531D-05	0.494D-06	0.801D-05	-0.184D-26	0.256D-12	0.149D-11	0.525D-12
-0.130D-05	0.448D-05	0.738D-05	-0.740D-27	-0.668D-13	0.525D-12	0.143D-11

COEFFICIENTS OF CORRELATION

0.100D+01	0.130D+00	-0.280D+00	0.519D+00	-0.676D+00	-0.609D+00	-0.152D+00
0.130D+00	0.100D+01	0.166D+00	-0.477D+00	-0.696D+00	0.582D-01	0.539D+00
-0.280D+00	0.166D+00	0.100D+01	0.322D+00	0.851D-01	0.793D+00	0.746D+00
0.519D+00	-0.477D+00	0.322D+00	0.100D+01	-0.115D-14	0.241D-14	0.146D-14
-0.676D+00	-0.696D+00	0.851D-01	-0.115D-14	0.100D+01	0.184D+00	-0.489D-01
-0.609D+00	0.582D-01	0.793D+00	0.241D-14	0.184D+00	0.100D+01	0.359D+00
-0.152D+00	0.539D+00	0.746D+00	0.146D-14	-0.489D-01	0.359D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-21 (cont'd)

RESIDUALS V
SPHERICAL

	V1(AUSNAT)				V2(NWL-9D)			V1 - V2		
21120	1.88	-1.89	-0.7	21120	-0.46	0.47	0.2	2.34	-2.36	-0.8
27070	0.54	-2.40	1.2	27070	-0.13	0.59	-0.3	0.68	-2.99	1.5
27090	-2.93	2.55	-0.7	27090	0.72	-0.63	0.2	-3.65	3.18	-0.8
27250	0.48	3.74	-0.1	27250	-0.12	-0.92	0.0	0.60	4.66	-0.2
27430	1.32	-1.83	-0.5	27430	-0.33	0.45	0.1	1.64	-2.28	-0.6
27490	-0.48	-4.98	-0.8	27490	0.12	1.23	0.2	-0.60	-6.21	-1.0
28050	-1.40	4.42	1.6	28050	0.35	-1.09	-0.4	-1.75	5.51	2.0

UNIT OF RESIDUALS (METERS)

Table 5-22

Transformation

AUSNAT -TO- NWL-9D (MOL'MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-122.41	-30.45	139.97	0.36	0.07	-0.52	0.35
± 1.40	± 1.37	± 1.40	± 0.91	± 0.24	± 0.25	± 0.25

VARIANCE - COVARIANCE MATRIX

SO2= 0.98

0.196D+01	-0.537D-01	0.684D-01	0.131D-06	0.648D-06	-0.109D-06	-0.122D-06
-0.537D-01	0.189D+01	-0.513D-01	0.435D-06	-0.218D-06	0.455D-07	0.246D-06
0.684D-01	-0.513D-01	0.196D+01	0.135D-06	0.761D-07	-0.409D-07	-0.674D-06
0.131D-06	0.435D-06	0.135D-06	0.823D-12	0.170D-28	0.270D-28	0.108D-28
0.648D-06	-0.218D-06	0.761D-07	0.416D-28	0.130D-11	0.256D-12	-0.668D-13
-0.109D-06	0.455D-07	-0.409D-07	0.925D-29	0.256D-12	0.149D-11	0.525D-12
-0.122D-06	0.246D-06	-0.674D-06	0.154D-28	-0.668D-13	0.525D-12	0.143D-11

COEFFICIENTS OF CORRELATION

0.100D+01	-0.279D-01	0.349D-01	0.103D+00	0.405D+00	-0.637D-01	-0.724D-01
-0.279D-01	0.100D+01	-0.267D-01	0.349D+00	-0.139D+00	0.271D-01	0.149D+00
0.349D-01	-0.267D-01	0.100D+01	0.107D+00	0.476D-01	-0.240D-01	-0.403D+00
0.103D+00	0.349D+00	0.107D+00	0.100D+01	0.164D-16	0.244D-16	0.993D-17
0.405D+00	-0.139D+00	0.476D-01	0.164D-16	0.100D+01	0.184D+00	-0.489D-01
-0.637D-01	0.271D-01	-0.240D-01	0.244D-16	0.184D+00	0.100D+01	0.359D+00
-0.724D-01	0.149D+00	-0.403D+00	0.993D-17	-0.489D-01	0.359D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN GREENWICH, EAST, AND CIO. THE ROTATIONS PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-23

Transformation

AUSNAT -TO- NWL-9D (VEIS-MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	ALPHA SECONDS	KSI SECONDS	ETA SECONDS
-122.41	-30.45	139.97	0.36	-0.59	0.10	0.20
± 1.40	± 1.37	± 1.40	± 0.91	± 0.19	± 0.29	± 0.25

VARIANCE - COVARIANCE MATRIX

SO2= 0.98

0.196D+01	-0.537D-01	0.684D-01	0.131D-06	-0.280D-06	0.163D-06	-0.584D-06
-0.537D-01	0.189D+01	-0.513D-01	0.435D-06	-0.260D-07	-0.210D-06	0.255D-06
0.684D-01	-0.513D-01	0.196D+01	0.135D-06	0.355D-06	0.519D-06	-0.257D-06
0.131D-06	0.435D-06	0.135D-06	0.823D-12	0.300D-28	0.539D-29	-0.100D-28
-0.280D-06	-0.260D-07	0.355D-06	0.108D-28	0.826D-12	0.441D-15	0.465D-15
0.163D-06	-0.210D-06	0.519D-06	0.462D-29	0.441D-15	0.198D-11	0.141D-12
-0.584D-06	0.255D-06	-0.257D-06	0.154D-28	0.465D-15	0.141D-12	0.142D-11

COEFFICIENTS OF CORRELATION

0.100D+01	-0.279D-01	0.349D-01	0.103D+00	-0.220D+00	0.827D-01	-0.350D+00
-0.279D-01	0.100D+01	-0.267D-01	0.349D+00	-0.209D-01	-0.109D+00	0.156D+00
0.349D-01	-0.267D-01	0.100D+01	0.107D+00	0.279D+00	0.264D+00	-0.155D+00
0.103D+00	0.349D+00	0.107D+00	0.100D+01	0.364D-16	0.422D-17	-0.927D-17
-0.220D+00	-0.209D-01	0.279D+00	0.364D-16	0.100D+01	0.345D-03	0.430D-03
0.827D-01	-0.109D+00	0.264D+00	0.422D-17	0.345D-03	0.100D+01	0.840D-01
-0.350D+00	0.156D+00	-0.155D+00	-0.927D-17	0.430D-03	0.840D-01	0.100D+01

Table 5-24

Transformation
SAD-69 -TO- NWL-9D (3 PARAMETER)
****

SOLUTION FOR 3 TRANSLATION PARAMETERS
(UNITS - METERS)

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS
-0.80376108D+02	-0.27416333D+00	-0.40306408D+02
± 2.56	± 2.56	± 2.56

VARIANCE - COVARIANCE MATRIX

MO2= 1.25

0.65366122D+01	0.0	0.0
0.0	0.65366122D+01	0.0
0.0	0.0	0.65366122D+01

COEFFICIENTS OF CORRELATION

0.10000000D+01	0.0	0.0
0.0	0.10000000D+01	0.0
0.0	0.0	0.10000000D+01

Table 5-24 (cont'd)

RESIDUALS V
SPHERICAL

V1(SAD-69)				V2(NWL-9D)				V1 - V2		
30009	7.78	-2.98	6.2	30009	-0.39	0.15	-0.3	8.17	-3.13	6.5
30010	8.00	-5.83	5.0	30010	-0.40	0.29	-0.2	8.40	-6.12	5.2
30012	-5.27	-5.46	-11.8	30012	0.26	0.27	0.6	-5.53	-5.73	-12.4
30022	7.99	-4.54	-10.6	30022	-0.40	0.23	0.5	8.39	-4.77	-11.1
30023	-8.06	-7.29	-2.1	30023	0.40	0.36	0.1	-8.47	-7.65	-2.2
30120	-11.24	-4.77	4.0	30120	0.56	0.24	-0.2	-11.80	-5.01	4.2
30121	-8.09	6.28	-0.0	30121	0.40	-0.31	0.0	-8.49	6.59	-0.0
30196	2.40	9.49	0.3	30196	-0.12	-0.47	-0.0	2.52	9.96	0.3
30209	1.72	12.38	3.5	30209	-0.09	-0.62	-0.2	1.81	13.00	3.6

UNIT OF RESIDUALS (METERS)

Table 5-25

Transformation

SAD-69 -TO- NWL-9D (BURSA)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-39.41	7.75	-37.36	-0.99	1.18	-0.90	0.16
±24.62	±11.76	± 7.56	± 1.16	± 0.86	± 0.33	± 0.26

VARIANCE - COVARIANCE MATRIX

SO2= 1.05

0.606D+03	0.215D+03	-0.224D+01	-0.277D-05	0.101D-03	-0.281D-04	0.103D-04
0.215D+03	0.138D+03	-0.189D+01	0.714D-05	0.394D-04	-0.985D-05	0.678D-05
-0.224D+01	-0.189D+01	0.572D+02	0.261D-05	-0.456D-06	-0.287D-05	-0.763D-05
-0.277D-05	0.714D-05	0.261D-05	0.134D-11	0.132D-27	-0.265D-27	-0.212D-27
0.101D-03	0.394D-04	-0.456D-06	-0.733D-26	0.174D-10	-0.434D-11	0.177D-11
-0.281D-04	-0.985D-05	-0.287D-05	0.193D-26	-0.434D-11	0.254D-11	-0.449D-12
0.103D-04	0.678D-05	-0.763D-05	-0.133D-26	0.177D-11	-0.449D-12	0.160D-11

COEFFICIENTS OF CORRELATION

0.100D+01	0.741D+00	-0.120D-01	-0.972D-01	0.986D+00	-0.716D+00	0.330D+00
0.741D+00	0.100D+01	-0.213D-01	0.525D+00	0.804D+00	-0.525D+00	0.455D+00
-0.120D-01	-0.213D-01	0.100D+01	0.298D+00	-0.144D-01	-0.238D+00	-0.796D+00
-0.972D-01	0.525D+00	0.298D+00	0.100D+01	0.274D-16	-0.143D-15	-0.144D-15
0.986D+00	0.804D+00	-0.144D-01	0.274D-16	0.100D+01	-0.653D+00	0.335D+00
-0.716D+00	-0.525D+00	-0.238D+00	-0.143D-15	-0.653D+00	0.100D+01	-0.222D+00
0.330D+00	0.455D+00	-0.796D+00	-0.144D-15	0.335D+00	-0.222D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-25 (cont'd)

RESIDUALS V
SPHERICAL

	V1(SAD-69)				V2(NWL-9D)			V1 - V2		
30009	6.42	1.68	6.3	30009	-0.32	-0.08	-0.3	6.75	1.76	6.7
30010	7.48	-1.33	4.4	30010	-0.37	0.07	-0.2	7.85	-1.40	4.6
30012	-6.83	-1.41	-7.1	30012	0.34	0.07	0.4	-7.17	-1.48	-7.4
30022	6.86	-0.39	-7.9	30022	-0.34	0.02	0.4	7.20	-0.41	-8.3
30023	-8.11	-5.30	-0.9	30023	0.41	0.27	0.0	-8.52	-5.57	-0.9
30120	-10.55	-3.14	3.7	30120	0.53	0.16	-0.2	-11.08	-3.30	3.9
30121	-10.19	1.24	5.6	30121	0.51	-0.06	-0.3	-10.70	1.30	5.9
30196	6.04	2.56	-1.7	30196	-0.30	-0.13	0.1	6.35	2.68	-1.7
30209	8.89	5.54	-2.6	30209	-0.44	-0.28	0.1	9.33	5.82	-2.7

UNIT OF RESIDUALS (METERS)

Table 5-26

Transformation

SAD-69 -TO- NWL-9D (MOL'MODELL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-77.78	-12.41	-49.53	-0.99	1.18	-0.90	0.16
± 4.73	± 8.65	± 4.25	± 1.16	± 0.86	± 0.33	± 0.26

VARIANCE - COVARIANCE MATRIX

SO2= 1.05

0.223D+02	-0.260D+02	-0.819D+01	0.265D-05	0.142D-04	-0.349D-05	0.144D-05
-0.260D+02	0.748D+02	0.199D+02	0.110D-05	-0.345D-04	0.861D-05	-0.356D-05
-0.819D+01	0.199D+02	0.180D+02	-0.475D-07	-0.100D-04	0.540D-05	-0.221D-05
0.265D-05	0.110D-05	-0.475D-07	0.134D-11	0.225D-27	-0.397D-28	0.198D-28
0.142D-04	-0.345D-04	-0.100D-04	0.206D-26	0.174D-10	-0.434D-11	0.177D-11
-0.349D-05	0.861D-05	0.540D-05	-0.556D-27	-0.434D-11	0.254D-11	-0.449D-12
0.144D-05	-0.356D-05	-0.221D-05	0.180D-27	0.177D-11	-0.449D-12	0.160D-11

COEFFICIENTS OF CORRELATION

0.100D+01	-0.635D+00	-0.408D+00	0.485D+00	0.720D+00	-0.463D+00	0.241D+00
-0.635D+00	0.100D+01	0.543D+00	0.110D+00	-0.956D+00	0.624D+00	-0.325D+00
-0.408D+00	0.543D+00	0.100D+01	-0.968D-02	-0.568D+00	0.798D+00	-0.411D+00
0.485D+00	0.110D+00	-0.968D-02	0.100D+01	0.466D-16	-0.215D-16	0.135D-16
0.720D+00	-0.956D+00	-0.568D+00	0.466D-16	0.100D+01	-0.653D+00	0.335D+00
-0.463D+00	0.624D+00	0.798D+00	-0.215D-16	-0.653D+00	0.100D+01	-0.222D+00
0.241D+00	-0.325D+00	-0.411D+00	0.135D-16	0.335D+00	-0.222D+00	0.100D+01

NOTE : THE POSITIVE AXES ARE TOWARDS MEAN
GREENWICH, EAST, AND CIO. THE ROTATIONS
PRINTED ARE ABOUT 3RD, 2ND, AND 1ST AXES RESPECTIVELY.

Table 5-27

Transformation

SAD-69 -TO- NWL-9D (VEIS MODEL)

****

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

DX METERS	DY METERS	DZ METERS	DELTA (X1.D+6)	ALPHA SECONDS	KSI SECONDS	ETA SECONDS
-77.78	-12.41	-49.53	-0.99	0.33	-0.48	-1.37
± 4.73	± 8.65	± 4.25	± 1.16	± 0.25	± 0.26	± 0.89

VARIANCE - COVARIANCE MATRIX

SO2= 1.05

0.223D+02	-0.260D+02	-0.819D+01	0.265D-05	-0.145D-05	-0.126D-05	-0.146D-04
-0.260D+02	0.748D+02	0.199D+02	0.110D-05	0.340D-05	0.310D-05	0.354D-04
-0.819D+01	0.199D+02	0.180D+02	-0.475D-07	-0.178D-05	0.196D-05	0.113D-04
0.265D-05	0.110D-05	-0.475D-07	0.134D-11	-0.217D-27	0.248D-29	-0.569D-27
-0.145D-05	0.340D-05	-0.178D-05	-0.229D-27	0.146D-11	0.142D-12	0.131D-11
-0.126D-05	0.310D-05	0.196D-05	-0.463D-28	0.142D-12	0.158D-11	0.163D-11
-0.146D-04	0.354D-04	0.113D-04	-0.101D-26	0.131D-11	0.163D-11	0.185D-10

COEFFICIENTS OF CORRELATION

0.100D+01	-0.635D+00	-0.408D+00	0.485D+00	-0.254D+00	-0.212D+00	-0.716D+00
-0.635D+00	0.100D+01	0.543D+00	0.110D+00	0.325D+00	0.286D+00	0.952D+00
-0.408D+00	0.543D+00	0.100D+01	-0.968D-02	-0.347D+00	0.368D+00	0.620D+00
0.485D+00	0.110D+00	-0.968D-02	0.100D+01	-0.156D-15	0.171D-17	-0.114D-15
-0.254D+00	0.325D+00	-0.347D+00	-0.156D-15	0.100D+01	0.933D-01	0.253D+00
-0.212D+00	0.286D+00	0.368D+00	0.171D-17	0.933D-01	0.100D+01	0.302D+00
-0.716D+00	0.952D+00	0.620D+00	-0.114D-15	0.253D+00	0.302D+00	0.100D+01

5.7 Summary

An attempt has been made to investigate distortions in various geodetic datums by comparing the differences in the residuals for each coordinate from adjustments for transformation parameters between the geodetic systems and the satellite system (NWL-9D). The residual differences were plotted and iso-residual-difference-lines were interpolated. In addition, "scale distortion maps" were plotted.

In case of the NAD 1927 Datum, Figs. 5-1 through 5-3 show quite remarkable patterns. It is not immediately clear whether the three-parameter, the four-parameter or the more "flexible" seven-parameter model is best suited as the transformation model. Moreover, since the residual patterns did not change drastically by increasing the number of transformation parameters from three to seven, it should be questioned if a similarity transformation model (up to seven parameters) or any other more general linear transformation model (up to 12 parameters: affine transformation) suffices to model the existing distortions.

However, the distortions seem to be largest in the Eastern and the Western parts of the United States, while they are smallest in the central area. This holds for both latitude and longitude. The distortions in height seem to be negligibly small. The scale distortion maps (Figs. 5-4 and 5-5) indicate large scale variations with systematic patterns. Basically, the scale seems to vary most in an east-west direction with a total difference from coast to coast of approximately 10 ppm. The scale distortion for the central area is between one and two parts per million. It thus becomes obvious that a (seven-parameter) similarity transformation is not sufficient for the transformation between the NAD 1927 and the NWL-9D

systems. In particular, a more detailed model for the scale should be used.

The residual patterns for the separate transformation of the Eastern part (Figs. 5-6 through 5-8) do not differ substantially from those discussed previously. Also, the scale factor of 1.84 ± 0.40 ppm is close to the one computed for all the points, i.e., 2.13 ± 0.27 ppm (Table 5-1). In contrast, the maps for the Western part of the NAD 1927 (Figs. 5-11 through 5-13) show distortions of reduced magnitudes as compared to those in Figs. 5-1 through 5-3. This is especially true for those transformations which include the scale parameter (Figs. 5-10 and 5-11). The scale factor is 5.95 ± 0.34 ppm (Table 5-1) which is much larger than all other scale factors computed so far. Fig. 5-11 which gives the residual differences for the seven-parameter transformation shows no systematic pattern anymore. In all cases, i.e., NAD 1927 EAST and NAD 1927 WEST, there are no significant distortions in height; but both parts differ in their orientation with respect to the NWL-9D system.

The results of the analysis of the Precise Traverse network (NAD - MR 1972) are presented in Figs. 5-12 through 5-15. None of these figures shows any systematic pattern. This was expected as it confirms the good quality of both measuring systems. The Doppler system NWL-9D is larger than the traverse system by 1.03 ± 0.16 ppm.

There are a number of factors to be considered in interpreting the distortion patterns found in this analysis. Here only a limited interpretation is attempted. A more complete investigation of the distortion pattern is in progress. The scale distortions as given in Fig. 5-4 and Fig. 5-5 are most likely caused by reducing the baselines to the geoid instead of

to the ellipsoid. There are striking similarities between the contours of the astrogeodetic geoid map and those of the scale distortion maps. The geoid to which the data of this analysis refer has zero height at Meades Ranch located in the central part of the United States. In this context, it should be mentioned again that the computed scale factor of Meades Ranch (Fig. 5-4) is between one and two ppm. If the amount of 1.03 ppm by which the NWL-9D system is too large is subtracted, there remains only a very small scale factor for the Meades Ranch area. This confirms that the scale factors of Fig. 5-4 may result from baselines reduced only to the geoid. It has been noted several times that the heights do not show a significant distortion. This becomes clear if it is recalled that the geodetic heights were computed by adding the astrogeodetic geoid undulation to the mean sea level heights. Thus the effect of the geoid has been taken into consideration. The distortions in latitude and longitude are a result of several factors which will have to be investigated in more detail.

The investigations with respect to other datums were limited due to lack of sufficient number of stations available.

REFERENCES

Anderle, R.J. 1975. Private communication. Naval Surface Weapons Center, Silver Springs, Maryland.

Badekas, J. 1969. "Investigations Related to the Establishment of a World Geodetic System," Report of the Department of Geodetic Science No. 124, The Ohio State University, Columbus.

Bomford, A.G. 1974. Private communication. Department of Minerals and Energy, Division of National Mapping, Canberra, Australia.

Bursa, M. 1962. "The Theory of the Determination of the Nonparallelism of the Minor Axis of the Reference Ellipsoid, Polar Axis of the Earth, and Initial Astronomical and Geodetic Meridians from Observations of Artificial Earth Satellites," translation from Studia Geophysica et Geodetica, Vol. 6, No. 2.

Bursa, M. 1967. "On the Possibility of Determining the Rotating Elements of Geodetic Reference Systems on the Basis of Satellite Observations," Studia Geophysica et Geodaetica, Vol. 11, No. 4, pp. 390-396.

Krakiwsky, E.J. and D.B. Thomson. 1974. "Mathematical Models for Terrestrial and Satellite Networks," The Canadian Surveyor, Vol. 28, No. 5.

Kumar, M. 1972. "Coordinate Transformation by Minimizing Correlations Between Parameters," Report of the Department of Geodetic Science No. 184, The Ohio State University, Columbus.

Leick, A., M. Arur and I.I. Mueller. 1975. "Western European Satellite Triangulation (WEST) Station Coordinates in the OSU WN14 System." Paper submitted to the XVIth General Assembly of the IUGG, Grenoble, France, August 18 - September 6.

Meade, B.K. 1974 and 1975. Private communication. National Oceanic and Atmospheric Administration, Rockville, Maryland.

Modenov, P.S. and A.S. Parkhomenko. 1961. Geometric Transformations, Volume 1: Euclidean and Affine Transformations. 1st edition. (Translated from Russian by M.B.P. Slater.) New York and London: Academic Press.

Molodenskii, M., V. Yeremeyev and M. Yurkina. 1962. Methods for Study of the External Gravitational Field and Figure of the Earth. Israel Program for Scientific Translations, Jerusalem.

Mueller, I.I., J.P. Reilly, M. Kumar, N. Saxena and T. Soler. 1973. "Global Satellite Triangulation and Trilateration for the National Geodetic Satellite Program (Solutions WN 12, 14 and 16)," Report of the Department of Geodetic Science No. 199, The Ohio State University, Columbus.

Mueller, I.I. 1974. "Global Satellite Triangulation and Trilateration Results," Journal of Geophysical Results, Vol. 79, No. 35.

Peat, R.P. 1975. Private communication. Defense Mapping Agency, Topographic Center, Washington, D.C.

Soler, T. 1976. "On Differential Transformations Between Cartesian and Curvilinear (Geodetic) Coordinate Systems," Report of the Department of Geodetic Science No. 236, The Ohio State University, Columbus.

Simmons, L.G. 1950. "How Accurate Is First-Order Triangulation?" Journal of the Coast and Geodetic Survey, No. 3.

Uotila, U.A. 1967. Unpublished lecture notes on Adjustment Computations, Department of Geodetic Science, The Ohio State University.

Vanicek, P. and D.E. Wells. 1974. "Positioning of Horizontal Geodetic Datums," The Canadian Surveyor, Vol. 28, No. 5.

Veis, G. 1960. "Geodetic Uses of Artificial Satellites," Smithsonian Contributions of Astrophysics, Vol. 3, No. 9, Smithsonian Institute, Washington, D.C.

Wells, D.E. and P. Vanicek. 1975. "Alignment of Geodetic and Satellite Coordinate Systems to the Average Terrestrial System," Bulletin Géodésique, No. 117.

Wolf, H. 1963. "Geometric Connection and Re-orientation of Three-Dimensional Triangulation Nets," Bulletin Géodésique, No. 68.

APPENDIX

Corrections to Transformation Parameters and Their Variances, As Published in [Mueller et al., 1973] and [Mueller, 1974].

Because of reasons discussed in this report, new tables are listed in this Appendix which supersede some of the tables given in earlier publications.

Table A-1 Shifts to the Geocenter (Solution WN 14)

Source	u_0, m	v_0, m	w_0, m	r_0, m
1. Dynamic comparison	16.9 ± 1.4	10.3 ± 1.5	-2.0 ± 2.0	19.1 ± 2.1
2. Geometric fit (equation (5))	23.2 ± 0.9	2.9 ± 0.8	-2.7 ± 1.2	23.4 ± 1.2
3. Weighted mean of 1 and 2	21.1 ± 1.2	4.7 ± 1.2	-2.4 ± 1.5	21.7 ± 1.8
4. JPL/DSN				25.9 ± 2.5

Supersedes Table 5.4-6 in [Mueller et al., 1973] and
Table 11 in [Mueller, 1974]

Table A-2

Relationships Between Various Solutions and the WN System (Other Solutions Minus WN14)

	Solution NWL 9D			Solution SAO III		
	s = 5000 n = 10 w = 1.4	s = 6000 n = 22 w = 8.0	All n = 32 w = 4.0	s = 6000 n = 48 w = 0.80	s = 9000 n = 22 w = 10.5	All n = 72 w = 4.0
Δu, m	13.8 ± 4.8	16.7 ± 1.2	16.3 ± 1.0	19.6 ± 1.2	14.1 ± 3.1	15.7 ± 1.4
Δv, m	11.2 ± 5.1	9.6 ± 1.2	10.1 ± 1.0	14.2 ± 1.2	16.1 ± 3.0	15.3 ± 1.4
Δw, m	-3.6 ± 5.7	-3.3 ± 1.2	-3.5 ± 1.1	-11.3 ± 1.3	-13.4 ± 3.4	-12.0 ± 1.5
$\Delta \times 10^6$	0.69± 0.70	0.30± 0.18	0.39± 0.16	0.60± 0.18	1.09± 0.45	0.97± 0.22
ω, "	0.86± 0.15	0.69± 0.05	0.72± 0.04	0.53± 0.05	0.32± 0.013	0.38± 0.05
ψ, "	0.00± 0.20	-0.16± 0.05	-0.15± 0.04	0.01± 0.06	-0.05± 0.14	0.02± 0.06
ε, "	0.13± 0.29	-0.17± 0.06	-0.17± 0.05	-0.18± 0.06	0.00± 0.13	0.00± 0.06
σ_0^2	1.31	0.93	1.16	1.14	1.06	0.97

s, type of stations considered; n, number of stations; w, weight factor equal to $\sigma_{0_i}^2/\sigma_{0_{WN14}}^2$.

Supersedes Table 5.4-1 in [Mueller et al., 1973] and Table 10 in [Mueller, 1974]

Table A-2 (cont'd)

Relationships Between Various Solutions and the WN System (Other Solutions Minus WN14)

	Solution GEM 6			Solution GSFC 73	Solution NGS Geometric Without Doppler	Solution NGS Geometric With Doppler
	s = 6000 n = 47 w = 0.75	s = 9000 n = 15 w = 3.50	All n = 75 w = 1.50	All n = 26 w = 22.0	s = 6000 n = 45 w = 2.25	s = 6000 n = 45 w = 2.75
Δu, m	20.7 ± 1.2	17.7 ± 2.8	18.0 ± 1.1	15.0 ± 2.2	-0.87 ± 1.03	18.8 ± 0.9
Δv, m	9.7 ± 1.2	10.5 ± 2.8	11.3 ± 2.4	11.3 ± 2.4	-7.6 ± 1.2	9.2 ± 0.9
Δw, m	0.7 ± 1.2	5.1 ± 2.8	3.9 ± 1.2	-1.9 ± 2.9	11.3 ± 1.3	-3.2 ± 1.0
$\Delta \times 10^6$	0.43± 0.18	0.70± 0.41	0.81± 0.17	0.74 ± 0.34	-2.29 ± 0.18	-2.33 ± 0.15
ω, "	0.11± 0.04	-0.05± 0.10	0.11± 0.04	-0.34 ± 0.08	0.11 ± 0.04	0.08 ± 0.04
ψ, "	0.09± 0.05	0.07± 0.12	0.17± 0.05	0.23 ± 0.11	-0.03 ± 0.05	-0.06 ± 0.04
ε, "	0.01± 0.05	0.07± 0.12	0.13± 0.04	0.29 ± 0.11	-0.04 ± 0.04	-0.07 ± 0.04
σ_0^2	1.01	1.32	1.06	1.10	1.05	1.14

s, type of stations considered; n, number of stations; w, weight factor equal to $\sigma_{0_1}^2/\sigma_{0_{WN14}}^2$.

Supersedes Table 5.4-1 in [Mueller et al., 1973] and

Table 10 in [Mueller, 1974]

Table A-3

Relationship Between Various Geodetic Datums and the WN System (Datum Minus WN 14)

Datum No.	Datum Name	No. of Stations	Δu (m)	Δv (m)	Δw (m)	ω (")	ψ (")	ϵ (")	$\Delta \times 10^6$
6	Australian Geodetic	3	118.4 ± 12.8	40.6 ± 16.4	-121.3 ± 18.0	1.03 ± 0.49	0.97 ± 0.48	-0.27 ± 0.59	-1.14 ± 1.83
16	European - 50 (W)	11	138.9 ± 30.9	111.1 ± 57.8	156.9 ± 29.0	-1.06 ± 1.37	0.10 ± 1.03	0.11 ± 1.60	-8.43 ± 4.34
	European - 50 (All Stations)	16	123.6 ± 24.9	146.1 ± 21.5	144.5 ± 23.9	-0.08 ± 0.67	0.03 ± 0.93	-0.41 ± 0.74	-6.06 ± 2.83
29	North American 1927 (W)	8	29.7 ± 16.0	-171.4 ± 11.0	-140.3 ± 16.6	0.22 ± 0.44	0.49 ± 0.48	-0.30 ± 0.52	-7.60 ± 1.66
	North American 1927 (E)	13	54.5 ± 19.5	-144.2 ± 11.5	-196.7 ± 11.6	0.96 ± 0.56	-0.03 ± 0.46	0.54 ± 0.40	2.26 ± 1.62
	North American (All Stations)	21	58.0 ± 6.6	-147.8 ± 7.8	-189.2 ± 9.2	0.89 ± 0.20	0.23 ± 0.19	0.38 ± 0.34	0.96 ± 0.98
41	South American 1969	10	55.0 ± 12.2	30.5 ± 10.1	41.6 ± 10.7	-0.59 ± 0.43	0.25 ± 0.31	-0.11 ± 0.34	6.66 ± 1.41

BURSA MODEL

Supersedes in parts Table 5.5-1 in [Mueller et al., 1973] and

Table 13 in [Mueller, 1974]

Table A-4

Relationship Between Various Geodetic Datums and the WN System (Datum Minus WN 14)

Datum No.	Datum Name	No. of Stations	Δu (m)	Δv (m)	Δw (m)	ω (")	ψ (")	ε(")	$\Delta \times 10^6$
6	Australian Geodetic	3	157.0 ± 3.2	59.2 ± 3.2	-131.2 ± 3.6	1.03 ± 0.49	0.97 ± 0.48	-0.27 ± 0.59	-1.14 ± 1.83
16	European-50 (W)	11	100.0 ± 5.3	125.8 ± 5.2	115.8 ± 5.3	-1.06 ± 1.37	0.10 ± 1.03	0.11 ± 1.60	-8.43 ± 4.34
	European-50 (All Stations)	16	99.4 ± 5.0	132.2 ± 5.0	116.4 ± 4.8	-0.08 ± 0.67	0.03 ± 0.93	-0.41 ± 0.74	-6.06 ± 2.83
29	North American 1927 (W)	8	20.6 ± 2.7	-139.3 ± 3.1	-179.6 ± 2.7	0.22 ± 0.44	0.49 ± 0.48	-0.30 ± 0.52	-7.60 ± 1.66
	North American 1927 (E)	13	30.8 ± 2.9	-141.4 ± 4.3	-174.7 ± 3.8	0.96 ± 0.56	-0.03 ± 0.46	0.54 ± 0.40	2.26 ± 1.62
	North American (All Stations)	21	31.6 ± 1.7	-142.1 ± 1.6	-177.3 ± 1.5	0.89 ± 0.20	0.23 ± 0.19	0.38 ± 0.34	0.96 ± 0.98
41	South American 1969	10	97.1 ± 4.0	13.4 ± 4.3	29.8 ± 4.1	-0.59 ± 0.43	0.25 ± 0.31	-0.11 ± 0.34	6.66 ± 1.41

M-Badekas Model

Note: This table is given only for reasons of comparison.